1. Consider the function \( f(x) = \frac{3}{5-2x} \).

(a) Is this function continuous on the interval \((-\infty, \infty)\)? Explain.
No. \( f \) is discontinuous at \( x = 2.5 \), where \( f \) is undefined (and has a vertical asymptote).

(b) Compute the average rate of change of \( f \) on \([2, 2.01]\).
\[
\frac{f(2.01) - f(2)}{2.01 - 2} = \left[ \frac{3}{5-2(2.01)} - \frac{3}{5-2(2)} \right] \cdot \frac{1}{.01} \approx 6.122
\]

(c) Using the limit definition of the derivative, compute \( f'(x) \).
\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
= \lim_{h \to 0} \frac{\frac{3}{5-2(x+h)} - \frac{3}{5-2x}}{h}
= \lim_{h \to 0} \frac{3(5-2x) - 3(5-2(x+h))}{h[(5-2(x+h))(5-2x)]}
= \lim_{h \to 0} \frac{15 - 6x - (15 - 6x - 6h)}{h(5-2(x+h))(5-2x)}
= \lim_{h \to 0} \frac{6h}{h(5-2(x+h))(5-2x)}
= \lim_{h \to 0} \frac{6}{(5-2x)^2}
\]

(d) Find the equation of the tangent line to \( f \) at \( x = 2 \).
We want \( y = mx + b \). \( m = f'(2) = \frac{6}{(5-2(2))^2} = 6 \), so \( y = 6x + b \).
[Note that this slope agrees well with our answer from (b) above.]
When \( x = 2, y = f(2) = \frac{3}{5-2(2)} = 3 \).
Thus, \( 3 = 6 \cdot 2 + b \), so \( b = -9 \) and we have \( y = 6x - 9 \).

2. Given that \( f(0) = 2, g(0) = 3, f'(0) = 5, g'(0) = 7, \) and \( f'(3) = \pi \) compute the following.

(a) \( h'(0) \) if \( h(z) = f(z)g(z) \)
\( h'(0) = f'(0)g(0) + f(0)g'(0) = (5)(3) + (2)(7) = 29 \)

(b) \( j'(0) \) if \( j(z) = \frac{f(z)}{g(z)} \)
\( j'(0) = \frac{f'(0)g(0) - f(0)g'(0)}{[g(0)]^2} = \frac{(5)(3) - (2)(7)}{3^2} = \frac{1}{9} \)

(c) \( k'(0) \) if \( k(z) = f(g(z)) \)
\( k'(0) = f'(g(0)) \cdot g'(0) = f'(3) \cdot (7) = (\pi)(7) = 7\pi \)
3. (a) Find \( \frac{dy}{dt} \) if \( y = t^5 + 5t + e^5 + \frac{t}{5} + \frac{5}{\sqrt{t}} + \ln(5t) + \arctan(5t) + \ln(5) + \sin 5 \).

\[
\frac{dy}{dt} = 5t^4 + (\ln 5)5t + 0 + \frac{1}{5} - 5t^{-2} + 5 \cdot \frac{-1}{5}t^{-6/5} + \frac{1}{5t} \cdot 5 + \frac{1}{1 + (5t)^2} \cdot 5 + 0 + 0
\]

\[
= 5t^4 + (\ln 5)5t + \frac{1}{5} - \frac{5}{t^2} - \frac{1}{t^{5/2}} + \frac{5}{1 + 25t^2}
\]

(b) Find \( \frac{dy}{dx} \) if \( y = \sqrt{x} \cos(7x^3) \).

\[
\frac{dy}{dx} = \frac{1}{3}x^{-2/3}\cos(7x^3) + \sqrt{x}(-\sin(7x^3)(21x^2)) = \frac{\cos(7x^3)}{3x^{2/3}} - 21x^{7/3}\sin(7x^3)
\]

(c) Find \( \frac{dy}{dz} \) if \( y = \frac{e^{z} + e^{-z}}{\tan 4 - 7z} \).

\[
\frac{dy}{dz} = \frac{e^z(\tan 4 - 7z) - (-7)(e^z + e^{-z})}{(\tan 4 - 7z)^2}
\]

(d) Find \( \frac{dy}{dr} \) if \( y = \tan(e^{r^2 \arcsin(5r)}) \).

\[
\frac{dy}{dr} = \sec^2(e^{r^2 \arcsin(5r)}) \cdot e^{r^2 \arcsin(5r)} \cdot \left[ r^2 \frac{1}{\sqrt{1 - 25r^2}} \cdot 5 + 2r \arcsin(5r) \right]
\]

(e) Find \( \frac{dy}{dx} \) if \( y^3 + yx^2 + x^3 = 3y^2 \). Here we use implicit differentiation.

\[
3y^2 \frac{dy}{dx} + \frac{dy}{dx} x^2 + 2xy + 2x = 6y \frac{dy}{dx}
\]

\[
3y^2 \frac{dy}{dx} + \frac{dy}{dx} x^2 - 6y \frac{dy}{dx} = -2xy - 2x
\]

\[
\frac{dy}{dx}(3y^2 + x^2 - 6y) = -2xy - 2x
\]

\[
\frac{dy}{dx} = -2xy - 2x = \frac{3y^2 + x^2 - 6y}{3y^2 + x^2 - 6y}
\]

(f) Find \( \frac{dy}{dt} \) if \( y = (1 + x^6)^{8x} \). Since we have \( x \) in the base and the exponent, we need logarithmic differentiation.

\[
\ln y = 8x \ln(1 + x^6)
\]

\[
\frac{1}{y} \frac{dy}{dx} = 8 \cdot \ln(1 + x^6) + 8x \cdot \frac{1}{1 + x^6} \cdot 6x^5
\]

\[
\frac{dy}{dx} = \left[ 8 \cdot \ln(1 + x^6) + \frac{48x^6}{1 + x^6} \right] \cdot y
\]

\[
\frac{dy}{dx} = \left[ 8 \cdot \ln(1 + x^6) + \frac{48x^6}{1 + x^6} \right] \cdot (1 + x^6)^{8x}
\]
4. Given the graph of \( f \), sketch a graph of \( f' \) and a graph of \( F \), an antiderivative of \( f \) such that \( F(0) = -1 \).

Note: The concave up portion on the left side of the graph of \( f \) is a perfect parabola, so its derivative \( (f') \) is linear; since you don’t know the equation for \( f \), your graph of \( f' \) may be concave up/down there.

5. Shown below is a graph of \( f' \) on its entire domain. The graph is NOT \( f \).

At which \( x \)-value(s)
(a) does \( f \) have a stationary point? \( c, f, h \)
(b) \( f \) increasing? \( c, g, h \)
(c) \( f \) increasing? \( b, d, g, i \)
(d) \( f \) concave up? \( a, b, d, g \)
(e) \( f \) concave down? \( b, d, g, i \)

On what interval(s) is
(a) \( f \) increasing? \( a, c \) \( \cup \) \( f, h \)
6. Solve the IVP \( y' = e^x - \sin x + 5 \) given that \( y(0) = 3 \). [Students in the 8:00 section may omit this problem.]
We antidifferentiate each side to obtain \( y(x) = e^x + \cos x + 5x + C \). To find \( C \), we let \( x = 0 \), meaning \( 3 = e^0 + \cos 0 + 5 \cdot 0 + C \), so \( C = 1 \) and our solution is \( y(x) = e^x + \cos x + 5x + 1 \).

7. Evaluate the following limits.
Throughout this solution, the symbol \( \star \) will stand for whatever notation your instructor prefers for using L’Hopital’s Rule on the indeterminate form 0/0; this may be \( \frac{0}{0} \) or \( \frac{L'}{H} \) or \( \frac{H'}{L} \) or “has the form ‘0/0’” or something else. The symbol \( \odot \) will serve the same purpose for the indeterminate form \( \infty/\infty \).

(a) \( \lim_{x \to \infty} \frac{x^2}{\ln x} \odot \lim_{x \to \infty} \frac{2x}{1/x} = \lim_{x \to \infty} 2x^2 = \infty \)

(b) \( \lim_{z \to 0} \frac{\sin(5z) - 5z}{z^3} \star \lim_{z \to 0} \frac{5\cos(5z) - 5}{3z^2} \star \lim_{z \to 0} \frac{-25\sin(5z)}{6z} \star \lim_{z \to 0} \frac{-125\cos(12z)}{6} = \frac{-125}{6} \)

(c) \( \lim_{x \to 0} \frac{e^x - 1}{x \cos x} = \frac{0}{1} = 0 \)

(d) \( \lim_{r \to 3} \frac{r^3 - 8}{r - 2} \star \lim_{r \to 3} \frac{3r^2}{1} = 12 \)

8. Consider the function \( f(x) = x^6 - 2x^3 \) on the interval \([-2, 2]\).

(a) Find the \( x \)- and \( y \)-coordinates of any and all local extrema and classify each as a local maximum, local minimum, or neither.
\[ f'(x) = 6x^5 - 6x^2 \]
Since \( f'(x) \) never fails to exist, we just need to solve \( f'(x) = 0 \).
\[ 0 = 6x^2(x^3 - 1) \]
\[ \Rightarrow x = 0, 1 \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( -2 \leq x &lt; 0 )</th>
<th>( 0 &lt; x &lt; 1 )</th>
<th>( 1 &lt; x \leq 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f )</td>
<td>negative</td>
<td>negative</td>
<td>positive</td>
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</table>

\( y \)-values: \( f(0) = 0 \), \( f(1) = -1 \)
So, \( f \) has a local minimum at \((1, -1)\); \((0, 0)\) is not a local extremum.

(b) Find the \( x \)- and \( y \)-coordinates of any and all global extrema and classify each as a global maximum or global minimum.
We check the \( y \)-values at the local extrema and the endpoints.
\( y \)-values: \( f(-2) = 80 \), \( f(1) = -1 \), \( f(2) = 48 \)
So, \( f \) has a global minimum at \((1, -1)\) and a global maximum at \((-2, 80)\).

(c) Find the \( x \)-coordinate(s) of any and all inflection points.
\[ f''(x) = 30x^4 - 12x \]
Since \( f''(x) \) never fails to exist, we just need to solve \( f''(x) = 0 \).
\[ 0 = 6x(5x^3 - 2) \]
\[ \Rightarrow x = 0, \sqrt[3]{0.4} \]

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<thead>
<tr>
<th>( x )</th>
<th>( x &lt; 0 )</th>
<th>( 0 &lt; x &lt; \sqrt[3]{0.4} )</th>
<th>( \sqrt[3]{0.4} &lt; x )</th>
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<tbody>
<tr>
<td>( f' )</td>
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<td>negative</td>
<td>positive</td>
</tr>
<tr>
<td>( f )</td>
<td>concave up</td>
<td>concave down</td>
<td>concave up</td>
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</table>
So, the \( x \)-values of the inflection points of \( f \) are \( x = 0 \) and \( x = \sqrt[3]{0.4} \).