1. Let $A = \begin{bmatrix} 1 & 1 \\ 2 & 4 \\ 4 & 16 \\ 5 & 25 \end{bmatrix}$ and $y = \begin{bmatrix} 13 \\ -12 \\ -5 \\ 21 \end{bmatrix}$. It's a fact that $y$ is not in $\text{Col}(A)$; you do not need to verify this.

1A. Find the least-squares solution of $Ax = y$. Show any and all matrices you used.

So we solve $A^TA = A^T \hat{y}$

obtaining $\begin{bmatrix} 46 & 198 & 74 \\ 198 & 898 & 410 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -7 \\ 0 & 1 & 2 \end{bmatrix} \Rightarrow x = \begin{bmatrix} -7 \\ 2 \end{bmatrix}$

1B. Find the projection vector $\hat{p}$ of $y$ onto the column space of $A$.

$\hat{p} = A\hat{x} = \begin{bmatrix} -5 \\ 0 \\ -6 \\ 15 \end{bmatrix}$

1C. Find the vector $z$ satisfying $p + z = y$.

so $z = \hat{y} - \hat{p} = \begin{bmatrix} 13 \\ -12 \\ -5 \\ 21 \end{bmatrix} - \begin{bmatrix} -5 \\ 0 \\ -6 \\ 15 \end{bmatrix} = \begin{bmatrix} 18 \\ -12 \\ -11 \\ 6 \end{bmatrix}$

1D. Find the distance from $y$ to $\text{Col}(A)$.

That's $\|z\|^2 = 324 + 144 + 81 + 36 = 585 \approx 23.84$

1E. Verify that $z$ is in $\text{Col}(A)^\perp$ by calculating the two appropriate dot products.

(you check that $\begin{bmatrix} -18 \\ -6 \\ -9 \\ 6 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 4 \\ 5 \end{bmatrix} = 0$

And $\begin{bmatrix} -18 \\ -6 \\ -9 \\ 6 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 16 \\ 25 \end{bmatrix} = 0$)

1F. The work done in this problem would be appropriate for solving which problem: (put a check next to the correct one)

Find the best fit curve of the form $y = \beta_1 x + \beta_2 x^2$ through the points $(1, 1), (2, 4), (4, 16)$ and $(5, 25)$.

Find the best fit curve of the form $y = \beta_3 + \beta_4 x$ through the points $(1, 13), (2, -12), (4, -5)$ and $(5, 21)$.

Find the best fit curve of the form $y = \beta_1 x + \beta_2 x^2$ through the points $(1, 13), (2, -12), (4, -5)$ and $(5, 21)$.

Find the best fit curve of the form $y = \beta_0 + \beta_1 x + \beta_2 x^2$ through the points $(1, 13), (2, 4, -12), (4, 16, -5)$ and $(5, 25, 21)$.