1. Let \( f \) be the function shown below. Which of the following is the best estimate of \( \int_0^5 f(x)\,dx \): -24, 9, 20, 38? Justify your answer.

Best estimate of the four choices is 20.
Each square in the grid has area 1 square unit.
The area must be positive (above x-axis).
The rectangle of width 6 and height 4 has area 24 square units and is a slight over-estimate.

2. Suppose that a car travels on a north-south road with northward velocity \( v(t) = 40 - 10t \) mph at time \( t \) hours.

a. Evaluate \( \int_0^5 v(t)\,dt \). Where is the car relative to its starting point at the end of 5 hours?

\[
\int_0^5 (40 - 10t)\,dt = \left[ 40t - 5t^2 \right]_0^5 = 200 - 125 = 75 \text{ miles}
\]

Start
\[ \begin{array}{c}
\text{v(t) = 40 - 10t} \\
40 - 10 \quad 80 \\
\hline
10 \quad 50 \\
\text{y(t) = y0 - 10t} \\
50 + 5 = 55 \text{ mi north of Start}
\end{array} \]

b. Let \( s(t) \) be the car's speed at time \( t \) (recall that \( s(t) = |v(t)| \)). Evaluate \( \int_0^5 s(t)\,dt \). What is the total distance traveled by the car at the end of 5 hours?

\[
\int_0^5 |40 - 10t|\,dt = \int_0^4 (40 - 10t)\,dt + \int_4^5 (10t - 40)\,dt
\]
\[
= \left[ 40t - 5t^2 \right]_0^4 + \left[ 5t^2 - 40t \right]_4^5
\]
\[
= 80 + 5 = 85 \text{ mi}
\]

(\text{Total distance traveled})
3. Use the fact that \( \int_1^4 t^2 \, dt = 21 \) along with the rules for manipulating integrals, to evaluate the following definite integrals:

\[
\text{10 points} \quad \text{a.} \quad \int_1^3 2r \, dr = - \int_1^4 r^2 \, dr = -21
\]

\[
\text{10 points} \quad \text{b.} \quad \int_1^4 (3x^2 - 1) \, dx = 3 \int_1^4 x^2 \, dx - \int_1^4 dx = 3 \left( \frac{x^3}{3} \right)
\]

\[
= 3 \left( \frac{4}{3} \right) - (4 - 1) = 6 - 3 = 3
\]

4. Let \( f(x) = x \) and \( A_f(x) = \int_0^x f(t) \, dt \). By computing the (signed) area represented by the integral, show that \( A_f(x) = \frac{x^2}{2} - 8 \) for all values of \( x \) (you need to consider \( x > 4 \), \( 0 \leq x \leq 4 \), and \( x < 0 \)). (Draw pictures!)

\[\text{25 points}\]

5. Evaluate the following definite integrals using the Fundamental Theorem of Calculus.

\[
\text{10 points} \quad \text{a.} \quad \int_0^4 \sqrt{x} \, dx = \frac{x^{3/2}}{3/2} \bigg|_0^4 = \frac{8}{3} - 8 = \frac{16}{3}
\]

\[
\text{10 points} \quad \text{b.} \quad \int_0^{\pi/2} \cos x \, dx = \sin x \bigg|_0^{\pi/2} = 1
\]