Math 206A: Winter 2012
Quiz 3: March 30

Correct answers accompanied by incorrect or incomplete work will not receive full credit. Good Luck!

1. Consider a function $f : \mathbb{R}^3 \rightarrow \mathbb{R}$. It has critical points $\vec{a}_1 = (2, 2, 1)$ and $\vec{a}_2 = (-3, -1, -1)$. Its Hessian matrix is

$$Hf = \begin{bmatrix} 2x^2y & 0 & x + y \\ 0 & y^2z & z \\ x + y & z & x \end{bmatrix}$$

(a) What is $\frac{\partial^2 f}{\partial x \partial z}$?

(b) Classify the critical points $\vec{a}_1$ and $\vec{a}_2$ as local maximum, local minimum, saddle points, or impossible to tell with the given information. Justify your answers.
2. Calculate \( \int_R (3x + 2y) \, dA \) where \( R \) is the region in the \( xy \)-plane bounded by the graphs of \( y = 2 \) and \( 2y = x^2 \).

3. Let \( \vec{F} : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \) be given by \( \vec{F}(x, y, z) = (xe^y) \hat{i} + (z \sin y) \hat{j} + (xy \ln z) \hat{k} \).
   
   (a) Calculate \( \text{div} \vec{F}(-3, 0, 2) \).

   (b) Calculate \( \text{curl} \vec{F} \).