Solutions

Math 206A: Winter 2012
Quiz 3: March 30

Correct answers accompanied by incorrect or incomplete work will not receive full credit. Good Luck!

Convergence Tests

- Geometric Series
- $n$th Term
- Comparison
- Integral
- $p$ Test
- Ratio
- Alternating Series
- Absolute Test

1. For each of the following series determine whether it converges or diverges. Carefully justify your answers.

(a) $\sum_{n=1}^{\infty} \frac{n}{3n+1}$

$$\lim_{n \to \infty} \frac{n}{3n+1} = \frac{1}{3} \neq 0$$

so series diverges by $n$th term test

(b) $\frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \frac{1}{81} + \cdots$

so this is a geometric series

$w/ r = -\frac{1}{3}$ and $|\frac{-1}{3}| < 1$

so the series converges by the geometric series test
2. Show that each of the following series converges. Then find good upper and lower bounds for each series.

(a) \[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} \]

- **Alternating**
- \[ C_n = \frac{1}{2n-1}, \quad C_{n+1} = \frac{1}{2(n+1)-1} = \frac{1}{2n+1} \]

- and \[ \frac{1}{2n-1} > \frac{1}{2n+1} \] for all \( n \geq 1 \)

- \[ \lim_{n \to \infty} \frac{1}{2n-1} = 0 \]

So series converge by **Alternating Series Test**.

\[ S_1 = \frac{(-1)^{2}}{2(1)-1} = \frac{1}{1} = 1 \quad S_2 = 1 + \frac{(-1)^{3}}{3} = \frac{2}{3} \]

So \[ \frac{2}{3} \leq \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} \leq 1 \]

(b) \[ \sum_{k=1}^{\infty} \frac{1}{(k+2)^2} \]

\[ \int_{1}^{\infty} \frac{1}{(x+2)^2} \, dx = \lim_{t \to \infty} \int_{1}^{t} \frac{1}{(x+2)^2} \, dx = \lim_{t \to \infty} \left[ -\frac{1}{x+2} \right]_{1}^{t} = \frac{1}{3} \]

Since the integral converges as shown above, the series converges by the Integral Test.

Also

\[ \frac{1}{3} \leq \sum_{k=1}^{\infty} \frac{1}{(k+2)^2} \leq \frac{1}{9} + \frac{1}{3} = \frac{4}{9} \]