MATH 205A,B - LINEAR ALGEBRA
WINTER 2013
QUIZ 9

NAME: ___________________________________________ Section:(Circle one)  A(1 : 10)  B(2 : 40)

Show ALL your work CAREFULLY.

(a) Let
\[ \vec{u}_1 = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, \quad \vec{u}_2 = \begin{bmatrix} 3 \\ -6 \\ 5 \end{bmatrix}, \quad \vec{u}_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}. \]

Determine whether the set \( S = \{ \vec{u}_1, \vec{u}_2, \vec{u}_3 \} \) is an orthogonal set. Justify your answer.

The set \( S \) is orthogonal if and only if each of the following dot products \( \vec{u}_1 \cdot \vec{u}_2, \vec{u}_2 \cdot \vec{u}_3, \vec{u}_3 \cdot \vec{u}_1 \) is zero. Since \( \vec{u}_1 \cdot \vec{u}_2 = (-1)(3) + (2)(-6) + (3)(5) = 0, \vec{u}_2 \cdot \vec{u}_3 = (3)(2) + (-6)(1) + (5)(0) = 0, \) and \( \vec{u}_3 \cdot \vec{u}_1 = (2)(-1) + (1)(2) + (0)(3) = 0, \) we conclude that \( S \) is an orthogonal set.

(b) Let \( \vec{y} = \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix} \) and \( \vec{u} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}. \) Find the projection of \( \vec{y} \) onto \( W = \text{Span}\{\vec{u}\}. \)

The projection of \( \vec{y} \) onto \( W \) is given by
\[
\text{proj}_W \vec{y} = \frac{\vec{y} \cdot \vec{u}}{\vec{u} \cdot \vec{u}} \vec{u} = \frac{(4)(1) + (-2)(-1) + (3)(2)}{1^2 + (-1)^2 + 2^2} \vec{u} = \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}.
\]

(c) Let \( \vec{y} \) and \( W \) be as in part (b). Find the shortest distance between \( \vec{y} \) and the line (through origin) \( W \).

The closest point in \( W \) to \( \vec{y} \) is \( \text{proj}_W \vec{y} \) so the shortest distance in question is the length
\[
\| \vec{y} - \text{proj}_W \vec{y} \| = \left\| \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix} - \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix} \right\| = \sqrt{5}.
\]

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