1. Let \( \mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \) and \( \mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \). Let \( S = \{ \mathbf{u}, \mathbf{v} \} \).

1A. Is \( S \) an orthogonal set? Explain your answer. NO, since \( \mathbf{u} \cdot \mathbf{v} = 1 \cdot 1 + 1 \cdot 2 + 1 \cdot 3 + 1 \cdot 4 = 10 \); if \( \mathbf{u} \perp \mathbf{v} \) then \( \mathbf{u} \cdot \mathbf{v} \) should be 0.

1B. Is \( S \) a linearly independent set? Explain. NO, since neither vector in \( S \) is a multiple of the other. (Alternate answers include showing \( \left( \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \right) \left( \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \right) = 0 \) has only the trivial solution \( x = 0 \).

1C. Let \( W \) be the subspace of \( \mathbb{R}^4 \) spanned by \( S \). What equations involving \( u \) and \( v \) does a vector \( w \) have to satisfy in order to be in \( W^\perp \)?

\[
\begin{align*}
\mathbf{w} \cdot \mathbf{u} &= 0 \\
\mathbf{w} \cdot \mathbf{v} &= 0
\end{align*}
\]

1D. Find a basis for \( W^\perp \). Show your work!

The two equations in 1C are represented by the matrix equation

\[
\begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & 2 & 3 & 4
\end{bmatrix}
\begin{bmatrix}
w_1 \\
w_2 \\
w_3 \\
w_4
\end{bmatrix} =
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

\[
\Rightarrow \mathbf{w} = w_3 \begin{bmatrix} 1 \\ -2 \\ 0 \\ 1 \end{bmatrix} + w_4 \begin{bmatrix} 2 \\ 0 \\ 3 \\ 0 \end{bmatrix}
\]

So a basis is \( \left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 3 \\ 0 \end{bmatrix} \right\} \).

2. Let \( u \) and \( v \) be as in problem 1.

2A. Find the distance from \( u \) to \( v \).

This is equal to \( \| \mathbf{u} - \mathbf{v} \| = \| \mathbf{v} - \mathbf{u} \| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3} \).

2B. Find a unit vector in the direction \( v \).

This is \( \frac{1}{\| \mathbf{v} \|} \mathbf{v} \), or \( \frac{1}{\sqrt{1^2 + 2^2 + 3^2}} \mathbf{v} \), or \( \frac{1}{\sqrt{14}} \mathbf{v} \).

3. Let \( u \) and \( v \) be as in problem 1.

3A. What is the total number of additions and multiplications required in computing \( \alpha(u \cdot v) \) if the result is computed in the order the parentheses mandate?

\( (\mathbf{u} \cdot \mathbf{v}) \) requires 4 multiplications, then 3 additions. One number results, and then multiplying by \( \alpha \) means one more multiplication.

**TOTALS:** 5 multiplies & 8 adds

3B. Repeat the question in 3A for \( (\alpha \mathbf{u}) \cdot \mathbf{v} \).

\( (\alpha \mathbf{u}) \) requires 4 multiplications. Forming the dot product of this with \( \mathbf{v} \) requires 4 more multiplications followed by 3 additions.

**TOTALS are** 8 multiplies & 8 adds