You may use the following facts in answering the questions below.

Suppose \( f(x) \) satisfies \( f(k) = a_k \) for \( k \geq 1 \). Suppose \( f \) is non-negative, decreasing and \( f(x) \to 0 \) as \( x \to \infty \).

Fact ONE: for any integer \( N > 0 \), we have

\[
\sum_{k=N+1}^{\infty} a_k \leq \int_{N}^{\infty} f(x) \, dx.
\]

Fact TWO: One antiderivative of \( \frac{x}{(x^2 + 4)^2} \) is \( -\frac{1}{2(x^2 + 4)} \).

1a. Consider the series \( \sum_{k=1}^{\infty} \frac{k}{(k^2 + 4)^2} \). Use the integral test to explain why this series must converge. Use good notation, especially where limits are involved.

Hint: (this was put on the board during the quiz): You may find it useful to find a general formula for \( \int_{N}^{\infty} \frac{x}{(x^2 + 4)^2} \, dx \)

1b. Suppose the series in (1a) converges to the number \( S \). Find a value of \( N \) for which the partial sum \( \sum_{k=1}^{N} \frac{k}{(k^2 + 4)^2} \) is within \( \epsilon = 0.0001 \) of \( S \). Show all your work.

Hint: (this was put on the board during the quiz): Remember that \( \sum_{k=1}^{\infty} a_k = \sum_{k=1}^{N} a_k + \sum_{k=N+1}^{\infty} a_k \)

1c. For your value of \( N \), use the LHS program to find the partial sum in (1b) to at least 8 places after the decimal point.