MATH 106A - CALCULUS II
WINTER 2007

QUIZ 8

NAME:

Show ALL your work CAREFULLY.

(a) Use the Integral Test to determine whether the following infinite series converges or diverges.

\[ \sum_{j=2}^{\infty} \frac{\ln j}{j^2} \]

Compare the series with the improper integral \( \int_{2}^{\infty} \frac{\ln x}{x^2} \, dx \). Note that for \( x \geq 2 \), the function \( \frac{\ln x}{x^2} \) is decreasing. Now,

\[
\int_{2}^{\infty} \frac{\ln x}{x^2} \, dx = \lim_{b \to \infty} \int_{2}^{b} \frac{\ln x}{x^2} \, dx = \left. \frac{\ln x \cdot (-x^{-1})}{-1} \right|_{2}^{b} - \int_{2}^{b} (-x^{-1}) \cdot \frac{1}{x} \, dx
\]

\[
= \lim_{b \to \infty} \left( \frac{\ln x}{x} \right) \bigg|_{2}^{b} - \ln 2 + \frac{1}{2}
\]

\[
= \ln 2 + \frac{1}{2} < \infty.
\]

Thus, the improper integral converges and by the Integral Test so does the infinite series.

(b) Use the Ratio Test to determine whether the following infinite series converges or diverges.

\[ \sum_{n=1}^{\infty} \frac{n^2}{2^n} \]

Here, \( a_n = \frac{a_{n+1}}{a_n} \). We have

\[
\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \lim_{n \to \infty} \frac{(n+1)^2}{2^{n+1}} \cdot \frac{2^n}{n^2}
\]

\[
= \lim_{n \to \infty} \left( \frac{n+1}{n} \right)^2 \cdot \frac{1}{2} = \frac{1}{2} < 1.
\]

By the Ratio Test, the series converges.

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