1. Let $b_1 = \begin{bmatrix} 3 \\ 9 \\ 7 \end{bmatrix}$, $b_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $p_1 = \begin{bmatrix} 6 \\ 27 \\ 26 \end{bmatrix}$, and $p_2 = \begin{bmatrix} 1 \\ 5 \\ 5 \end{bmatrix}$. Let $\mathcal{B} = \{b_1, b_2\}$ and let $\mathcal{P} = \{p_1, p_2\}$.

It’s a fact that $\mathcal{B}$ and $\mathcal{P}$ are bases for the same subspace $H$ of $\mathbb{R}^3$; you do not need to prove this.

1A: Find $[b_1]_\mathcal{P}$.

1B: Find the change-of-basis matrix $M$ from $\mathcal{B}$ to $\mathcal{P}$.

2. Let $K = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 3 & 3 & 8 & 0 \\ 4 & 6 & 7 & 0 \\ 0 & 0 & 5 & 2 \end{bmatrix}$.

Make good use of the 0’s in the matrix $K$ to find the determinant of $K$ using the “cofactor expansion” method we’ve used in class. Show all the intermediate steps and results. (You can check your final answer on your calculator, of course).

3. Let $S = \begin{bmatrix} 1 & -3 \\ 4 & 8 \end{bmatrix}$. Find the characteristic polynomial of $S$, and the eigenvalues of $S$. 