Math 105 Test Two

Name: ______________________

1. (1 mark each)
   a. \( f(x) = e^x \cos x \) find \( f'(x) \)
   b. \( f(x) = b^x \ln b \) find \( \int f \)
   
   c. \( f(x) = \tan(2x + 3) \) find \( f'(x) \)
   d. \( f(x) = \ln x \) find \( \int f \)
   
   e. \( f(x) = \frac{\cos x}{1 + \sin^2 x} \) find \( \int f \)
2. (5 marks) $f(x)$ is a differentiable function and $g(x) = f(ax)$ where $a \neq 0$.
Use the limit definition of derivative to show that $g'(x) = af'(ax)$. 

3. (5 marks) Evaluate the following limits without the use of a calculator.
   a. \( \lim_{x \to \infty} \frac{\sin x}{x} \)  
   b. \( \lim_{x \to 0} \frac{2x+3}{5-4x} \)
   c. \( \lim_{x \to -\infty} \frac{e^x}{x^2} \)  
   d. \( \lim_{x \to 0^+} x^2 \ln x \)
   e. \( \lim_{x \to 0} (1 + x)^{1/x} \)
4. Use calculus to answer the following questions.
   a. (4 marks) Differentiate \( y = x + \cos(xy) \) with respect to \( x \).
b. (1 mark) A car drives on an elliptic track given by the graph of the equation $5x^2 - 6xy + 5y^2 = 16$ where direction north is coincident with the positive $y$ axis. At what points on the track is the car headed north or south?
5. Use calculus to answer the following questions.

a. (4 marks) Where on \([0, \pi]\) does \(f(x) = e^x \cos x\) achieve a maximum value?
b. (1 mark) A wire of length $L$ is to be cut into two pieces. One piece will be used to form a square; the other, to form a circle. How should the wire be cut to maximize the sum of the areas of the pieces?