Quiz #5

Name: ________________________________

1. Consider \( \sum_{k=1}^{\infty} \frac{e^k}{2k+1} \).

(a) Find the first three terms of the sequence \( \{s_n\} \). You do not have to simplify terms.

**Solution:**

\[
\begin{align*}
s_1 &= \frac{e}{3} \\
s_2 &= \frac{e}{3} + \frac{e^2}{5} \\
s_3 &= \frac{e}{3} + \frac{e^2}{5} + \frac{e^3}{7}
\end{align*}
\]

(b) Does the series converge or diverge? Justify your answer.

**Solution:** The series **diverges** by the \( n \)th term test:

\[\lim_{k \to \infty} \frac{e^k}{2k+1} = +\infty,\]

by L'Hôpital’s rule.

2. Does \( \sum_{j=0}^{\infty} \frac{2^{j+1}}{3^j} \) converge or diverge? Justify your answer.

**Solution:** Write out a few terms of the series:

\[
\sum_{j=0}^{\infty} \frac{2^{j+1}}{3^j} = 2 + \frac{4}{3} + \frac{8}{9} + \frac{16}{27} + \ldots
\]

This is a geometric series with \( r = \frac{2}{3} \) and \( a = 2 \). Since \( |r| = \frac{2}{3} < 1 \), the series **converges** and moreover,

\[
\sum_{j=0}^{\infty} \frac{2^{j+1}}{3^j} = \frac{2}{1 - 2/3} = \frac{2}{1/3} = 6.
\]