Math 205 (Winter 2016)
Test 2 (50 points)

Name: ____________________________________________

• Check that you have eight questions on three pages.

• Show all your work to receive full credit for a problem. Points will be taken off if you do not show how you arrived at your answer, even if the answer is correct.

• Please keep your explanations brief; be clear and to the point. Points will be taken off for incorrect or irrelevant statements.

1. (6 points) Let $A$ and $B$ be $6 \times 6$ matrices, with $\det A = -10$ and $\det B = 5$. Use properties of determinants to compute:

   (a) $\det 3A$

   (b) $\det (A^T B^{-1})$
2. (7 points) Let \( A = \begin{bmatrix} 5 & 1 \\ 0 & 5 \end{bmatrix} \). This matrix \( A \) has only one eigenvalue which is 5.

(a) Find a basis for the eigenspace corresponding to the eigenvalue 5.

(b) Is the matrix \( A \) diagonalizable? Explain.
3. (9 points) Let \( \vec{v} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix} \) and \( \vec{y} = \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix} \).

(a) Find a unit vector in the direction of \( \vec{v} \).

(b) Find the distance between \( \vec{v} \) and \( \vec{y} \).

(c) Let \( L = \text{Span}\{\vec{v}\} \). Compute the orthogonal projection of \( \vec{y} \) onto \( L \).
4. (4 points) Suppose \( \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\} \) is an orthogonal set of non-zero vectors in \( \mathbb{R}^4 \). Is the set \( \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\} \) a basis for \( \mathbb{R}^4 \)? Explain.

5. (6 points) Suppose \( \{\vec{u}, \vec{v}\} \) is a basis of the eigenspace corresponding to the eigenvalue 0 of a \( 5 \times 5 \) matrix \( A \).

(a) Is \( \vec{w} = \vec{u} - 2\vec{v} \) an eigenvector of \( A \)? If so, find the corresponding eigenvalue. If not, explain why.

(b) Find the dimension of \( \text{Col} \ A \).
6. (8 points) Let \( C = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -3 \\ 1 & 2 & 1 \end{bmatrix} \).

(a) Find a basis for \( \text{Col} \ C \).

(b) Let \( \vec{x} = \begin{bmatrix} -3 \\ -11 \\ 7 \end{bmatrix} \) be a vector in \( \text{Col} \ C \). Find the coordinates of \( \vec{x} \) with respect to the basis you found in part (a).
7. (5 points) Let \( B = \begin{pmatrix} -2 & 0 & -9 \\ 0 & 3 & 0 \\ 0 & 5 & 6 \end{pmatrix} \). Find all the eigenvalues of \( B \).

8. (5 points) Let \( W = \text{Span} \left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} \right\} \). Is \( \begin{bmatrix} 2 \\ 6 \\ 4 \\ 0 \end{bmatrix} \) in \( W^\perp \)? Explain.