Math 105: Review for Exam II

1. Find $dy/dx$ for each of the following.

   (a) $y = x^2 + 2^x + e^x + 2^x + \ln 2 + \ln(2x) + (\ln 2)x + \tan 2$

   (b) $y = \sqrt{x} \cdot \tan(5x)$

   (c) $y = \ln(\tan(2\cos(x^2)))$

   (d) $y = \frac{x + e^x}{\cos 4 + \sin^3(6x)}$
2. Consider the curve defined by \( x^3 + y^3 = \frac{9}{2} xy \) (known as the Folium of Descartes).

(a) Find \( \frac{dy}{dx} \).

(b) Verify that the point \((1,2)\) is on the curve above.

(c) Find the equation of the tangent line at the point \((1,2)\).

3. Evaluate the following limits.

(a) \( \lim_{x \to 1} \frac{x^3 - 1}{7 - 7x} \)

(b) \( \lim_{x \to 0} \frac{1 - \cos(2x)}{3^x} \)

(c) \( \lim_{x \to 0} \frac{1 - \cos(4x)}{5x^2} \)

(d) \( \lim_{x \to \infty} \frac{x^2}{2^x} \)
4. Consider the function $f(x) = x^4 e^x$ with domain all real numbers.

(a) Find the $x$-value(s) of all roots (x-intercepts) of $f$.

(b) Find the $x$- and $y$-value(s) of all critical points and identify each as a local max, local min, or neither.

(c) Find the $x$- and $y$-value(s) of all global extrema and identify each as a global max or global min.

(d) Find the $x$-value(s) of all inflection points.

(e) Sketch $f$. 
5. How would your answers to the previous question change if the domain of \( f \) were \([-10, 10]\)?

6. (Please omit this problem.) Use the Intermediate Value Theorem to explain why \( f(x) = x^3 - 4x^2 + 5 \) must have a root somewhere on the interval \([1, 2]\).

7. Find an antiderivative of \( y = \frac{5}{\sqrt{1 - 9x^2}} + x^3 + \cos(2x) + e^3 \).

8. You are planning to build a box-shaped aquarium with no top and with two square ends. Your budget is $288. If the glass for the sides costs $12 per square foot and the opaque material for the bottom costs $3 per square foot, what dimensions will maximize the volume? Be sure to show how you know you have found the maximum.