MATH 106A,B - CALCULUS II WINTER 2014

QUIZ 8

NAME:

Show ALL your work CAREFULLY.

(a) Determine whether the following infinite sequence \( \{a_n\} \) converges or diverges. If it converges, find its limit.

\[
a_n = \left(1 + \frac{2}{n}\right)^{n/2}
\]

Suppose \( L = \lim_{n \to \infty} a_n \). Then

\[
\ln L = \lim_{n \to \infty} \frac{n}{2} \left(1 + \frac{2}{n}\right) = \lim_{n \to \infty} \left(1 + \frac{2}{n}\right) \frac{(\frac{2}{n})}{\left(\frac{2}{n}\right)} = \frac{L}{L'} \lim_{n \to \infty} \left(-\frac{2}{n^2}\right) = 1.
\]

It follows that \( L = e \) so \( \lim_{n \to \infty} a_n = e \).

(b) Evaluate the following infinite series if it converges.

\[
\sum_{j=1}^{\infty} \left[ 2 \left(\frac{1}{3}\right)^j - \left(\frac{2}{5}\right)^j \right]
\]

Note that this is NOT a geometric but it is the difference between two (convergent) geometric series. Now, we have

\[
\sum_{j=1}^{\infty} \left[ 2 \left(\frac{1}{3}\right)^j - \left(\frac{2}{5}\right)^j \right] = \sum_{j=1}^{\infty} 2 \left(\frac{1}{3}\right)^j - \sum_{j=1}^{\infty} \left(\frac{2}{5}\right)^j = \frac{2}{3 - \frac{1}{3}} - \frac{2}{5 - \frac{2}{5}} = \frac{1}{3}.
\]

Date: March 17, 2014.