1. Consider the geometric series \( \sum_{k=0}^{\infty} 5 \cdot \left( \frac{2}{3} \right)^k \)

1A. Explicitly write out the first four members of the sequence of partial sums as simplified fractions, that is, find \( S_0, S_1, S_2, \) and \( S_3 \) each in the form \( a/b \) where \( a/b \) is in “lowest terms”.

\[
\begin{align*}
S_0 &= 5 \\
S_1 &= 5 + 5 \cdot \frac{2}{3} = 5 + \frac{10}{3} = \frac{15}{3} + \frac{10}{3} = \frac{25}{3} \\
S_2 &= 5 + 5 \cdot \frac{2}{3} + 5 \cdot \frac{4}{9} = \frac{25}{3} + \frac{20}{9} = \frac{25 \cdot 3}{9} + \frac{20}{9} = \frac{95}{9} \\
S_3 &= 5 + 5 \cdot \frac{2}{3} + 5 \cdot \frac{4}{9} + 5 \cdot \frac{8}{27} = S_2 + 5 \cdot \frac{4}{9} = \frac{95}{9} + \frac{40}{27} = \frac{285 + 40}{27} = \frac{325}{27}
\end{align*}
\]

\[
S_0 = 5 \quad S_1 = \frac{25}{3} \quad S_2 = \frac{95}{9} \quad S_3 = \frac{325}{27}
\]

1B. Use the formula developed in class to find \( S_{22} \).

\[
S_{22} = \frac{S - S \left( \frac{2}{3} \right)^{22}}{1 - \frac{2}{3}} = \frac{14.99866343...}{1 - \frac{2}{3}} = \frac{325}{27}
\]

1C. How many terms are added together in 1B? (21? 22? or 23?)

\[
S_{22} = a_0 + a_1 + \ldots + a_{22} = \text{sum of 23 terms}
\]

1D. Does this geometric series converge? If so, to what? Show any formulas involved in answering this.

Yes it converges, because \( r = \frac{2}{3} \) satisfies \( |r| < 1 \).

We know that it converges to \( \frac{a}{1 - r} = \frac{5}{1 - \frac{2}{3}} = \frac{5}{\frac{1}{3}} = 15 \)

2. Find \( r \) for which \( \sum_{k=0}^{\infty} 5r^k \) converges to 40; express \( r \) as a fraction in lowest terms. If there is no such \( r \), explain why.

We need to find \( |r| < 1 \) satisfying \( \frac{5}{1 - r} = 40 \)

So \( 5 = 40 - 40r \)

\(-35 = -40r \)

\( r = \frac{35}{40} = \frac{7}{8} \) which is indeed < 1