NAME:

Show ALL your work CAREFULLY.

(a) Find the limit of the following sequence or explain why it does not exist.

\[ a_n = n(1 - \cos(1/n)) \]

First rewrite \( a_n = \frac{1 - \cos(1/n)}{1/n} \). Now, we apply l’Hôpital’s rule so that

\[
\lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{\sin(1/n) \cdot (-1/n^2)}{(-1/n^2)} = \lim_{n \to \infty} \sin(1/n) = 0.
\]

(b) Determine whether the following infinite series converges or diverges. If it converges, evaluate the infinite series. [Hint: write the first few terms of the series.]

\[
\sum_{n=3}^{\infty} 2 \left( \frac{e}{\pi} \right)^n
\]

By writing the first few terms of the series, it is easy to see that this is a geometric series with common ratio \( r = e/\pi < 1 \) and the first term of the series is \( a = 2(e/\pi)^3 \). Since \( |r| < 1 \), it follows that the series converges and so

\[
\sum_{n=3}^{\infty} 2 \left( \frac{e}{\pi} \right)^n = \frac{a}{1 - r} = \frac{2(e/\pi)^3}{1 - e/\pi}.
\]