1. Consider the sequence \( a_1, a_2, a_3, \ldots \) which begins as follows:

\[
\begin{aligned}
1 & \quad 2 \quad 1 \quad 3 \quad 1 \quad 4 \quad 1 \quad 5 \\
2' & \quad 3' \quad 1' \quad 4' \quad 1' \quad 5' \\
3' & \quad 2' \quad 1' \quad 4' \quad 1' \quad 5' \\
4 & \quad 3 \quad 1 \quad 5 \quad 1 \\
4' & \quad 3' \quad 1' \quad 5' \\
5 & \quad 4 \\
5' & \quad 4' \\
6 & \quad 5
\end{aligned}
\]

1A: If this pattern continues, what are the next two terms of the sequence?

\( \frac{1}{6}, \frac{6}{7} \)

1B: Find the limit \( L \) of this sequence or explain why it doesn't converge.

There is **no** limit here. The odd numbered terms are approaching 0 while the terms in the even numbered positions are approaching 1. So there is no common value that **ALL** terms are approaching.

(Note well: this is really just an observation that a proof would be needed to show for any \( \epsilon > 0 \) \( \frac{\ln 2}{5} < \epsilon \)).

2. Consider the sequence whose \( k \)th term is \( a_k = \frac{2k^2 + \ln k}{5k + 3k^2} \). Show how to use l'Hopital's rule and \( \frac{2x^2 + \ln x}{5x + 3x^2} \) to determine the limit of this sequence.

\[
\begin{aligned}
limit_{k \to \infty} a_k &= \lim_{k \to \infty} \frac{2k^2 + \ln k}{5k + 3k^2} \\
&= \lim_{x \to \infty} \frac{2x^2 + \ln x}{5x + 3x^2} \\
&= \lim_{x \to \infty} \frac{4x + \frac{1}{x}}{5 + 6x} \\
&= \lim_{x \to \infty} \frac{4 + \left( \frac{7}{x^2} \right)}{6} \\
\end{aligned}
\]

This has the indeterminate form \( \infty/\infty \)

This is not indeterminate form \( \frac{0}{0} \) as \( x \to 0 \) but \( \frac{2}{3} \)

3. Consider the sequence whose \( k \)th term is \( a_k = \frac{3k + 5}{k} \).

3A. What is the limit \( L \) of this sequence? \( 3 \)

3B. Let \( \epsilon = 0.04 \). By examining a table of values of \( a_k \), find \( N \) such that if \( n \geq N \) then \( |a_n - L| < \epsilon \) yet \( |a_{N-1} - L| \) is not less than \( \epsilon \). (That is, find the "best" \( N \)). Give both \( N \) and \( |a_{N-1} - L| \).

\( N = 126 \)

\( |a_{N-1} - L| = 0.04 \)

(since \( |a_{125} - L| = |3.04 - 3| = 0.04 \) and \( 0.04 \) is not \( \epsilon \)).

3C. BONUS. Find the "best" \( N \) given any arbitrary \( \epsilon > 0 \) (show all your work).

We need \( n \geq N \) to satisfy

\[
\begin{aligned}
|a_n - L| &< \epsilon \\
|\frac{3n + 5}{n} - 3| &< \epsilon \\
|\frac{3n + 5 - 3n}{n} - 3n| &< \epsilon
\end{aligned}
\]

So choose \( N \) the smallest integer for which \( \frac{3}{\epsilon} < N \).