Math 105: Review for Exam II

1. Find \( dy/dx \) for each of the following.
   (a) \( y = x^2 + 2^x + e^2 + e^{2x} + \ln 2 + \ln(2x) + \arctan 2 \)

   (b) \( y = \sqrt{x} \cdot \arctan(5x) \)

   (c) \( y = \ln(\tan(2^{\cos(x^2)}) \)

   (d) \( y = \frac{x + e^\pi}{\cos 4 + \sin^5(6x)} \)

2. Consider the curve defined by \( x^3 + y^3 = \frac{9}{2} xy \) (known as the Folium of Descartes).
   (a) Find \( dy/dx \).

   (b) Verify that the point \( (1,2) \) is on the curve above.

   (c) Find the equation of the tangent line at the point \( (1,2) \).
3. Find the following.

(a) an antiderivative of \( y = \frac{5}{\sqrt{1 - 9x^2}} + x^3 + \cos(2x) + e^3 \)

(b) \( \tan(\arccos x) \) (rewritten as an algebraic expression - no trigonometric functions)

4. Consider the function \( f(x) = x^4 e^x \) with domain all real numbers.

(a) Find the \( x \)-value(s) of all roots (\( x \)-intercepts) of \( f \).

(b) Find the \( x \)- and \( y \)-value(s) of all critical points and identify each as a local max, local min, or neither.

(c) Find the \( x \)- and \( y \)-value(s) of all global extrema and identify each as a global max or global min.
(d) Find the $x$-value(s) of all inflection points.

(e) Sketch $f$.

5. How would your answers to the previous question change if the domain of $f$ were $[-10, 10]$?

6. Find the $c$ specified in the Mean Value Theorem for $f(x) = \sqrt{x}$ on the interval $[1, 9]$.

7. Does the Extreme Value Theorem apply to $f(x) = x^2$ on the interval $[1, 5]$?
8. Can the Intermediate Value Theorem be applied to show that $f(x) = x + \ln x$ has a root on the interval $[1, e]$?

9. You are planning to build a box-shaped aquarium with no top and with two square ends. Your budget is $288. If the glass for the sides costs $12 per square foot and the opaque material for the bottom costs $3 per square foot, what dimensions will maximize the volume? Be sure to show how you know you have found the maximum.