March 12, 2009
Mathematics 206a
Multivariable Calculus
Examination #2

(10) I. Let C be the curve parametrized by
\[ \vec{\gamma}(t) = \left( t, \frac{2\sqrt{2}}{3} t^\frac{3}{2}, \frac{t^2}{2} \right) \]
starting at \( t = 0 \) and ending at \( t = 2 \). Give an equation for the plane that passes through \( \vec{\gamma}(1) \) and is perpendicular to the tangent to \( C \) at that point.

(10) II. For the vector field \( \vec{F}(x, y, z) = (xy, -\sin z, 1) \), compute

A. \( \text{div} (\vec{F}) = \)

B. \( \text{curl} (\vec{F}) = \)
(10) III. Find the equation of the plane tangent to the graph of \( z = x^2 + y^4 + e^{xy} \) at the point \((1, 0, 2)\). 

(10) IV. Derivatives

A. Suppose \( f(x, y, z) = (x + z + y, x^2) \) and \( a = (1, 1, 0) \). Calculate the total derivative of \( f \) at \( a \).

B. Suppose \( g: \mathbb{R} \to \mathbb{R}^3 \) with rule \( g(t) = (6t^2, 3t^3, t) \) and \( f: \mathbb{R}^3 \to \mathbb{R} \) with rule \( xyz \). Use the Chain Rule to calculate \( (f \circ g)'(1) \).
(10) V. Find the critical points of $f(x, y, z) = x^2 + y^2 + z^2 - x - y - 2z + 1$ and determine whether they are local maxima, local minima, or saddle points.

(10) VI. If $f: \mathbb{R}^3 \rightarrow \mathbb{R}$ has rule $f(x, y, z) = x^2 e^{-yz}$, calculate the rate of change of $f$ at $(1, 0, 0)$ in the direction parallel to the vector $\mathbf{i} + \mathbf{j} + \mathbf{k}$. 
(10) VII. Suppose $f: \mathbb{R}^3 \to \mathbb{R}$ with rule $f(x, y, z) = \sin(x + 2y + 3z)$.

A. Give $p_1(x)$, the first Taylor Polynomial of $f$ at $(0, 0, 0)$.

B. Give $p_2(x)$, the second Taylor Polynomial of $f$ at $(0, 0, 0)$.

(10) VIII. If $S$ is the solid below the surface $z = 4 - x^2 - y^2$ and above the $x$-$y$ plane, set up but do not evaluate an iterated integral whose value is the triple integral $\iiint_S f(x, y, z) \, dV$. 
(10) IX. Suppose $C$ is the closed parametrized by $f(t) = (3\cos t, 3\sin t)$ starting at $t = 0$ and ending at $t = 2\pi$.

A. If $F : \mathbb{R}^2 \rightarrow \mathbb{R}$ with $F(x, y) = x + y$, Compute $\int_C F \, dL$.

B. If $F : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ with $F(x, y) = xi + yj$. Compute $\int_C \vec{F} \cdot d\vec{x}$. 
(10) X. Evaluate the iterated integral and draw the region in the plane determined by the limits on the integral.

$$\int_{0}^{\pi} \int_{\cos x}^{\pi} y \sin x \ dy \ dx$$.