Please show your work.

1. Evaluate the following:
   
   a) \( \int \sin^3(x)\cos^4(x) \, dx \)

   b) \( \int x^2 \ln(x) \, dx \)

2. Determine if the following integrals converge or diverge. If the integral converges, evaluate the integral.

   a) \( \int_1^\infty \frac{1}{\sqrt{x}} \, dx \)
b) \( \int_{-1}^{2} \frac{dx}{x^3} \)

c) \( \int_{0}^{\pi/2} \sec(\theta) \, d\theta \)

3. Use comparison to show whether the following improper integral converges or diverges. Justify your answer.

\[ \int_{1}^{\infty} \frac{1}{\sqrt{x}(1 + x)} \, dx \]
4. a) Find the 2^{nd}-order Taylor Polynomial for \( f(x) = \sqrt{x} \) based at \( x_0 = 4 \).

b) Use your results from part (a) to estimate \( \sqrt{5} \).

5. Taylor’s Theorem says that \( |f(x) - P_n(x)| \leq \frac{K_{n+1}}{(n+1)!} |x - x_0|^{n+1} \) for all values of \( x \) in an interval containing \( x_0 \). Using this result, what is the largest possible error that could have occurred in your estimate of \( \sqrt{5} \).
6. Does the following improper integral converge or diverge? Justify your answer. If it converges, evaluate the integral.

\[
\int_{3}^{\infty} \frac{1}{x(x-1)} \, dx
\]

7. Does the following improper integral converge or diverge? Justify your answer. If it converges, evaluate the integral.

\[
\int_{0}^{\infty} xe^{-x} \, dx
\]

8. Evaluate: \[ \int \frac{dx}{(x^2 + 1)^{3/2}} \]. Note: \((x^2 + 1)^{3/2} = (\sqrt{x^2 + 1})^3\)