Question 1) Evaluate the following integrals

a) \[ \int \sin^4 x \cos x \, dx. \]

b) \[ \int \sin^2 x \cos^2 x \, dx. \]
Question 2) Evaluate the integral

\[ \int \frac{x + 2}{x^2 - 9} \, dx. \]
Question 3a) Evaluate the integral
\[ \int x \cos(3x) \, dx. \]
without using the table of integrals.

3b) Evaluate the integral
\[ \int \frac{\sqrt{x^2 - 1}}{x} \, dx. \]
Question 4) Let $T$ be the number of minutes that it takes a randomly selected person to solve a certain puzzle. The probability density function of $T$ is

$$f(x) = \begin{cases} 
3/x^4 & \text{if } x \geq 1 \\
0 & \text{if } x < 1 
\end{cases}$$

a) What is the probability that $T > 2$?

b) The expected number of minutes (the mean) that it takes for a random person to solve the puzzle is given by $\int_{-\infty}^{\infty} x f(x) \, dx$. Find the expected number of minutes it takes a random person to solve the puzzle.
Question 5) Consider the function $f(x) = \sqrt{x}$.

a) Find $P_4(x)$, the 4th order Taylor polynomial, of $f(x)$ centered at $x = 1$.

b) Use $P_4(x)$ to find an estimate for $\sqrt{2}$.

c) Use Taylor’s Theorem to approximate the error of your estimate from part (b) on the interval $[1, 2]$. Recall that error bounds for estimates using a Taylor Polynomial $P_n(x)$ may be determined using:

$$|f(x) - P_n(x)| \leq \frac{K_{n+1}}{(n+1)!}|x - x_0|^{n+1},$$

where $K_n$ is a constant such that $|f^{(n+1)}(x)| \leq K_{n+1}$ for all $x$ in $[1, 2]$. 
Question 6) Determine whether or not the improper integral

\[ \int_{e}^{\infty} \frac{dx}{x(\ln x)^3} \]

converges. If it does, evaluate it.
Question 7) Determine whether or not the improper integral
\[
\int_{1}^{\infty} \frac{dx}{x^3 + \sqrt{x}}
\]
converges using a comparison test.
Useful Trigonometric Identities:

- $\sin^2 x + \cos^2 x = 1$
- $\tan^2 x + 1 = \sec^2 x$
- $\sin(2x) = 2 \sin x \cos x$
- $\cos(2x) = \cos^2 x - \sin^2 x$
- $\sin^2 x = \frac{1}{2}(1 - \cos(2x))$
- $\cos^2 x = \frac{1}{2}(1 + \cos(2x))$