1. (12 pts. each) Evaluate the following integrals.

(a) $$\int x^2 \ln x \, dx$$

(b) $$\int \frac{x^3}{\sqrt{9 - x^2}} \, dx$$
(c) \( \int \sec^4 x \tan^2 x \, dx \)

(d) \( \int \frac{2x + 3}{x(x + 1)^2} \, dx \)
2. (13 pts.) Consider \( \int_0^1 \frac{1}{x^3 + x} \, dx \). Use a comparison to determine if this integral converges or diverges?
3. (15 pts.) Consider the function \( f(x) = \sqrt{1 - x} \).

(a) Find \( P_3(x) \), the 3\(^{rd} \) order Taylor Polynomial, of \( f(x) \) centered at \( x = 0 \). Simplify your answer as much as possible, in other words, fractional coefficients must be in lowest terms.

(b) Use \( P_3(x) \) to find an estimate for \( \sqrt{0.5} \).

(c) Use Taylor’s Theorem to approximate the error of your estimate from part (b) on the interval \([0, \frac{1}{2}]\). Recall that error bounds for estimates using a Taylor Polynomial \( P_n(x) \) may be determined using:

\[
|f(x) - P_n(x)| \leq \frac{K_{n+1}}{(n + 1)!} |x - x_0|^{n+1}.
\]
4. (12 pts. each) Suppose that the time (in hours) it takes to finish this exam is given by the probability density function

\[ f(x) = \begin{cases} \frac{k}{x^2} e^{-x^3} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases} \]

(a) Find the value of \( k \) that ensures \( f \) is a probability density function.

(b) One way to find an “average time” is to compute the median. The median is the number \( N \) such that \( \int_0^N f(x) \, dx = \frac{1}{2} \), i.e., exactly half of the class will finish the test in \( N \) hours or less.

Find \( N \). (Be sure to use your value for \( k \) from part (a) in this integral.)
Useful Formulas

Trigonometric Identities

• $\sin^2 \theta + \cos^2 \theta = 1$

• $\sec^2 \theta = 1 + \tan^2 \theta$

• $\sin(2\theta) = 2 \sin \theta \cos \theta$

• $\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$

• $\sin^2 \theta = \frac{1}{2}(1 - \cos(2\theta))$

• $\cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta))$

Some antiderivative rules:

• $\int \sec \theta \, d\theta = \ln|\sec \theta + \tan \theta| + C$

• $\int \csc \theta \, d\theta = \ln|\csc \theta - \cot \theta| + C$