NAME:

Show ALL your work CAREFULLY.

(a) Evaluate the improper integral
\[ \int_0^1 \frac{1}{\sqrt{(3x-1)}} \, dx. \]

Using a simple substitution with \( u = 3x - 1 \), the indefinite integral
\[ \int \frac{1}{\sqrt{(3x-1)}} \, dx = \int u^{-1/3} \frac{1}{3} \, du \]
\[ = \frac{1}{3} u^{2/3} + C = \frac{1}{2} (3x - 1)^{2/3} + C. \]

Now, the improper integral can be written as
\[ \int_0^1 \frac{1}{\sqrt{(3x-1)}} \, dx = \int_0^{1/3} \frac{1}{\sqrt{(3x-1)}} \, dx + \int_{1/3}^1 \frac{1}{\sqrt{(3x-1)}} \, dx \]
\[ = \lim_{a \to 1/3} \int_0^a \frac{1}{\sqrt{(3x-1)}} \, dx + \lim_{b \to 1/3} \int_b^1 \frac{1}{\sqrt{(3x-1)}} \, dx \]
\[ = \lim_{a \to 1/3} \left[ \frac{1}{2} (3a - 1)^{2/3} - \frac{1}{2} (-1)^{2/3} \right] + \lim_{b \to 1/3} \left[ \frac{1}{2} (3 - 1)^{2/3} - \frac{1}{2} (3b - 1)^{2/3} \right] \]
\[ = \frac{1}{2} (\sqrt[3]{4} - 1). \]

(b) Use comparison to determine whether the improper integral
\[ \int_2^\infty \frac{1}{x + x^4} \, dx \]
converges or diverges. Justify your answer.

For \( x \geq 2, x + x^4 > x^4 \) so that \( 0 < \frac{1}{x + x^4} < \frac{1}{x^4} \). It follows that
\[ 0 < \int_2^\infty \frac{1}{x + x^4} \, dx < \int_2^\infty \frac{1}{x^4} \, dx < \int_1^\infty \frac{1}{x^4} \, dx < \infty. \]
The last inequality is the result of the \( p \)-test since \( p = 4 > 1 \). Thus, by comparison, we have shown that the improper integral \( \int_2^\infty \frac{1}{x + x^4} \, dx \) converges.