1. Consider the following vectors in $\mathbb{P}_4$: Let $\mathbf{v}_1 = 3x^4 + 5x^2 + 6$, $\mathbf{v}_2 = x^4 + 2x^2 + 3$, and $\mathbf{v}_3 = 5x^4 + 7x^2 + 4$. Let $\mathbf{b}$ be the polynomial $49x^4 + 73x^2 + 58$.

1A) In terms of the unknowns $\alpha_1, \alpha_2$ and $\alpha_3$, what system of equations do you need to set up to determine if $\mathbf{b}$ can be written as a linear combination $\alpha_1\mathbf{v}_1 + \alpha_2\mathbf{v}_2 + \alpha_3\mathbf{v}_3 = \mathbf{b}$?

1B) Now determine the values of $\alpha_1$, $\alpha_2$ and $\alpha_3$ or explain why there are none. Show any RREF’d matrices you use.

1C) Without setting up any equations or finding any RREF’s, give a quick reason why $\mathbf{c} = 5x^3 + 7x^2 + 11$ is obviously not a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$. 

* * * This quiz CONTINUES on the OTHER SIDE * * *
2. Let $H$ be the set of functions in $F$ whose graphs are completely above the horizontal line $y = 2$.

2A. Is $v_1 = x^2 + 4$ in $H$? You can draw a graph of $v_1$ (copy it here) to explain your answer.

2B. Is $v_2 = 2 + \sin x$ in $H$? You can draw a graph of $v_2$ (copy it here) to explain your answer.

2C. Explain informally why $H$ is closed under the vector addition of $F$ (hint: think about “addition of $y$-coordinates”) or give an explicit counter example.

2D. Explain informally why $H$ is closed under scalar multiplication (hint: think about “multiplying $y$-coordinates by any arbitrary scalar”) or give an explicit counter example that shows $H$ is not closed under scalar multiplication.