1. Suppose that \( T(x, y) = x^2 + 3y^2 - x \) is the temperature (in degrees Celsius) at the point \((x, y)\) (where \(x\) and \(y\) are in meters). If you are standing at the point \((-2, 1)\) and proceed in the direction \(-\hat{i} - \hat{j}\), will the temperature be increasing or decreasing at the moment you begin? At what rate? Include units in your answer.

\[
\begin{align*}
T_x(x, y) &= 2x - 1 \\
T_x(-2, 1) &= -5 \\
T_y(x, y) &= 6y \\
T_y(-2, 1) &= 6
\end{align*}
\]

\[\nabla T(-2, 1) = -5\hat{x} + 6\hat{j}\]

\[
\mathbf{v} = -\hat{i} - \hat{j}
\]

\[
\|\mathbf{v}\| = \sqrt{2}
\]

\[
\frac{\nabla T(-2, 1)}{\|\mathbf{v}\|} = \frac{-5\hat{x} + 6\hat{j}}{\sqrt{2}}
\]

\[
\mathbf{T}_u(-2, 1) = (-5\hat{x} + 6\hat{j}) \cdot \left(\frac{-\hat{i} - \hat{j}}{\sqrt{2}}\right) = \frac{-5}{\sqrt{2}} - \frac{6}{\sqrt{2}} = -\frac{11}{\sqrt{2}}
\]

\[\Rightarrow \quad \text{Temp is decreasing at a rate of } \frac{1}{\sqrt{2}} \text{ °C/m} \]

2. Find the equation of the plane tangent to the surface \(x^2 + 2xy + 4y - 3 = z^2\) at the point \((-4, 1, 3)\).

\[
f(x, y, z) = x^2 + 2xy + 4y - 3 - z^2
\]

\[
f_x(x, y, z) = 2x + 2y \quad \rightarrow \quad f_x(-4, 1, 3) = -6
\]

\[
f_y(x, y, z) = 2x + 4 \quad \rightarrow \quad f_y(-4, 1, 3) = -4
\]

\[
f_z(x, y, z) = -2z \quad \rightarrow \quad f_z(-4, 1, 3) = -6
\]

\[
-6(x + 4) - 4(y - 1) - 6(z - 3) = 0
\]
3. Suppose that as you move away from the point \( (2, 0) \), \( f(x, y) \) increases most rapidly in the direction \( 0.6i + 0.8j \) and the rate of increase in this direction is 5. Find \( \nabla f(2, 0) \).

\[ \nabla f(2, 0) \text{ points in the direction of most rapid increase, so in the direction of } 0.6i + 0.8j. \]

\[ \| \nabla f(2, 0) \| = \text{rate of greatest increase} \]

so \[ \| \nabla f(2, 0) \| = 5 \]

Note \[ \| 0.6i + 0.8j \| = \sqrt{(0.6)^2 + (0.8)^2} = 1 \]

so \[ \nabla f(2, 0) = 5 (0.6i + 0.8j) = 3i + 4j \]