(10) I. Define any two of these terms. Use a complete, mathematically correct sentence for each definition.

- cycle
- the alternating group on $n$ letters
- left coset of a subgroup of a group $\langle G, * \rangle$

A. 

B. 

(4) II. Complete the following statement of Lagrange’s Theorem:

If $G$ is a finite group and $H$ is a subgroup of $G$, then ...

__________________________________________________________

__________________________________________________________
(16) III. Give examples of:

A. A homomorphism from $\mathbb{Z}_3$ to $\mathbb{Z}_6$ whose kernel is $\{0\}$.

B. A non-trivial homomorphism from $\mathbb{Z}_8$ to $\mathbb{Z}_6$

C. A non-abelian group with 12 elements.

D. A cyclic group that is isomorphic to the direct product of two of its non-trivial subgroups.
(21) IV. Fill in the blanks:

A. One generator for \( Z_8 \times Z_9 \) is _______________________.

B. The order of \((1, 2, 3)(2, 3, 4)\) in \( S_4 \) is _______________________.

D. The number of left cosets of \( \langle 5 \rangle \) in \( Z_{10} \) is ____________________.

E. The order of the group \( S_6 \) is ____________________.

G. Express \((1, 4, 2, 5, 3, 7) \in S_7\) as a product of transpositions __________________.

H. The order of the factor group \((Z_{11} \times Z_6)/\langle 1, 3 \rangle \) is ____________________.

I. The subgroup of \((Z_4 \times Z_8)/\langle 0, 2 \rangle \) generated by \((3, 3) + \langle 0, 2 \rangle \) has ______________________ elements.

(9) V. If \( \langle G, * \rangle \) is an abelian group with identity \( e \), define \( H = \{ x \in G \mid x * x = e \} \).
If \( G = Z_2 \times Z_4 \), what is \( H \)?
VI. Suppose \( \phi : G \to G' \) is a homomorphism and that \( \ker (\phi) = \{e\} \). Let \( x, y \in G \) and suppose \( \phi(x) = \phi(y) \). Prove that \( x = y \). You may use all the other results we have proven for homomorphisms, such as \( \phi(e) = e' \), \( (\phi(y))^{-1} = \phi(y^{-1}) \), etc. [Hint: Consider \( \phi(xy^{-1}) \).]

VII. If \( H \) is a subgroup of \( G \), \( a \) an element of \( G \), and \( aH = Ha \), prove that if \( h \) is an arbitrary element of \( H \), then \( aha^{-1} \) is an element of \( H \).
(10) VIII. Draw a regular pentagon and label its vertices with the integers 1, 2, 3, 4, and 5. The symmetries of the pentagon form a group of order 10, sometimes denoted $D_5$ and called the 5th dihedral group. Use cycle notation to express each of these symmetries as a permutation of the vertices of the pentagon.
IX. TRUE OR FALSE? (Don't guess! The number of incorrect responses will be subtracted from the number of correct ones.)

_____ 1. $\mathbb{Z}_5 \times \mathbb{Z}_{25}$ is a cyclic group.

_____ 2. The number of elements in any subgroup of a finite group $G$ divides the number of elements in $G$.

_____ 3. Every permutation is a cycle.

_____ 4. The direct product of abelian groups is always abelian.

_____ 5. $S_6$ has no cyclic subgroups.

_____ 6. The composition of two permutations of a set $A$ is always a permutation of $A$.

_____ 7. Every left coset of a subgroup of a group $G$ is also a subgroup of $G$.

_____ 8. Every abelian group of order 8 contains a cyclic subgroup of order 8.

_____ 9. Every cycle is a permutation

_____ 10. Every finite group of prime order is cyclic