1. Suppose $\det(A) = 600$ where $A = \begin{bmatrix} 5 & y & 2 & 4 \\ 0 & 0 & w & 0 \\ 0 & 10 & 9 & 7 \\ 0 & 0 & 6 & 3 \end{bmatrix}$.

1A. Find $A_{23} = \begin{bmatrix} 5 \\ y \\ 0 \\ 0 \end{bmatrix}$.

1B. Find $(A_{23})$. $5 \cdot 10 \cdot 3 = 150$ (since $A_{23}$ is upper-triangular).

1C. Find $w$ (notice there are a lot of zeros in its row).

$\det(A) = 600 = -w \cdot \begin{vmatrix} 5 & y \\ 0 & 0 \end{vmatrix} = -w \cdot 150$ \implies $w = \frac{600}{150} = 4$.

1D. Does finding $\det(A)$ by expansion across either the first row or down the second column tell you what $y$ is? Explain. No. In either expansion, the only term involving $y$ is $-y \cdot \begin{vmatrix} 0 & 0 \\ 0 & 3 \end{vmatrix} = -y \cdot 0$, so the value of $y$ doesn't matter in either expansion and so they are of no help in finding $y$.

1E. Bonus: Find $y$.

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2. Let $B = \begin{bmatrix} a & b & c \\ 5 & 6 & 7 \\ m & n & p \end{bmatrix}$; suppose $\det(B) = 3$.

Find the determinant of each of the following matrices, and under each matrix write the reason/rule/fact about determinants of matrices you used to find the det. (e.g., "swapping rows changes the sign of the det" or "the determinant of the derivative of a matrix is the matrix of its integral" (this second fact is nonsense).

2A. $\det(B) = \begin{vmatrix} a + 20 & b + 24 & c + 28 \\ 5 & 6 & 7 \\ m & n & p \end{vmatrix}$; the det is: $3$.

2B. $\det(B) = \begin{vmatrix} a & 5 & m \\ b & 6 & n \\ c & 7 & p \end{vmatrix}$; the det is: $3$ since for any matrix $M$, $\det(M) = \det(M^T)$.

2C. $\begin{vmatrix} a & b & c \\ m & n & p \end{vmatrix}$; the det is: $3$.

2D. $\begin{vmatrix} a & b & c \\ 5 & 6 & 7 \\ m & n & p \end{vmatrix}$; the det is: $0$ since this matrix is row equivalent to $\begin{vmatrix} 5 & 6 & 7 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$ and this has $0$ determinant. But you know two of the three of the determinants involved.

2E. $B^2 = \begin{vmatrix} 24 \\ 20 \end{vmatrix}$; the det is: $3$.

$\det(B) = \frac{1}{2} \det(26)$ because $B = \begin{vmatrix} 24 \\ 20 \end{vmatrix}$.

$\det(26) = 12$.

2F. $B^{-1}$

Hint for 2F: You know $BB^{-1} = I_3$, so both sides of this equation must have the same determinant. But you know two of the three of the determinants involved.

3. Suppose $U = \begin{bmatrix} 4 & 6 & 2 \\ 0 & 3 & 7 \\ 0 & 0 & 5 \end{bmatrix}$. Suppose $C$ is row equivalent to $U$ and the elementary row operations which produce $U$ from $C$ are:

a) one row swap.

b) a multiple of one row is added to another and this is done 3 times.

c) one row operation involves dividing a row by 4.

d) one row operation involves multiplying a row by 5.

3A. What is the determinant of $C$?

$$\det(C) = (-1)(1)(1)(4)(-1)(-1)(-1)(-1) = -4 \cdot \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$$

3B. Explain why do NOT need to know the exact order in which these row operations are performed in order to find $\det(C)$.

(See 3A)