Define a function $\phi: Z_5 \times Z_5 \to Z_5 \times Z_5$ by $\phi(x, y) = (x - y, x + y)$

A. What is $\phi((a, b) + (c, d))$?

B. What is $\phi((a, b)) + \phi((c, d))$?

C. Let $(x, y) \in Z_5 \times Z_5$. Prove that $\phi$ is onto by finding an element $(a, b)$ in $Z_5 \times Z_5$ such that $\phi(a, b) = (x, y)$. Note that your formulas for $a$ and $b$ will be in terms of $x$ and $y$.

D. Determine $\ker \phi$, the kernel of $\phi$, i.e. all elements $(a, b)$ such that $\phi(a, b) = (0, 0)$

E. What do your answers to A-D prove about $\phi$?