1. Suppose that \( \mathbf{u}(\mathbf{x}) = (f(x), g(x), h(x), k(x)) \) where the four component functions \( f, g, h, k \) are real-valued and \( \mathbf{x} = (x, y, z) \). Suppose further that \( x = r^2 + s^3 + t, \ y = s^2 t^5, \) and \( z = e^{8t} + r^4 \), and \( \mathbf{m}(r, s, t) \) is the vector-valued function with these three component functions \( x, y \) and \( z \).

1A. Find \( J(\mathbf{m}) \); simplify all entries.

1B. how many rows, and columns, does \( J\mathbf{u} \) have? rows = \[\] columns = \[\].

1C. Let \( u_N \) be \( \mathbf{u} \circ \mathbf{m} \), and let \( u_{N,2} \) be the second component function of \( u_N \). Find and simplify to the extent possible a formula for \( \partial u_{N,2}/\partial t \). (Hint: Fill out just enough of \( J\mathbf{u} \) so that with your answer to 1A you can find the answer to this problem)

2. Let \( \mathbf{P} = (3, 4) \) and \( \mathbf{Q} = (15, 9) \) be two points in the plane. Let \( f(x, y) = x^3 + xy \). Let \( \mathbf{u} \) be a unit vector pointing in the direction from \( \mathbf{P} \) to \( \mathbf{Q} \).

2A. Find the directional derivative of \( f \) in the direction \( \mathbf{u} \) at the point \( \mathbf{P} \). Show all your work. Express your final answer as a decimal to five places after the decimal point.

2B. What is the approximation to this partial derivative, using \( h = 0.1 \) and the difference quotient \( (f(\mathbf{P} + h\mathbf{u}) - f(\mathbf{P}))/h \)? Show all your work to five places after the decimal point.