1. Let \( f(x) = x^{4/3} = \sqrt[3]{x^4} \). You do not need to verify this, but in powers of \((x - 1)\), the second degree Taylor polynomial for \( f(x) \) is \( P_2(x) = 1 + \frac{4}{3}(x - 1) + \frac{2}{9}(x - 1)^2 \), and \( f^{(3)}(x) = -\frac{8}{27}x^{-5/3} \).

(1A) On the interval \((0.9, 1.2)\), what is the error guarantee for this polynomial? Show all your work, including the “error guarantee formula”.

\[
\text{first we need } K \text{ satisfying } K \geq |f^{(3)}(x)| \text{ on } (0.9, 1.2)
\]

The graph of \( f^{(3)}(x) \):

\[
\frac{0.36}{3!} (1.2 - 1)^3 = \frac{0.36 \times (2)^3}{6} = \frac{0.00258}{6} = 0.00043
\]

now the guarantee is

\[
|f^{(3)}(0.9)| = |-0.353191| \text{ to two decimals}
\]

we choose \( K = 0.36 \)

(1B) What is the actual error at \( x = 0.9 \) (show how you found it).

\[
\frac{\frac{\delta}{\delta x} f(0.9) - P_2(0.9)}{\delta x} = \frac{0.8689404...}{0.8688888...} \approx 0.0000512
\]

2. Consider the improper integral \( \int_{12}^{\infty} (x + 13)^{-3/2} \, dx \).

(2A) Show that it converges, and find out to what. As we’ve done in class, make a table using at least four well-chosen \( B \) values that obviously supports your conclusion. Use good notation throughout, including the limit process involved.

\[
\int_{12}^{\infty} (x + 13)^{-3/2} \, dx \text{ is by definition}
\]

\[
\lim_{B \to \infty} \left( \int_{12}^{B} (x + 13)^{-3/2} \, dx \right)
\]

\[
= \lim_{B \to \infty} \left( \int_{12}^{B} \left( \frac{-2}{\sqrt{B+13}} + \frac{2}{12+13} \right) \, dx \right)
\]

\[
= \lim_{B \to \infty} \left( \frac{-2}{\sqrt{B+13}} + \frac{2}{12+13} \right) = \frac{2}{5}
\]

\[
\begin{array}{c|c|c|c}
B & 100 & 0.2118 & \\
1000 & 0.3976 & \\
10000 & 0.39936 & \\
\downarrow & & & \\
0 & 0.4 & \\
\end{array}
\]

(2B) BONUS: Find \( B \) to several digits for which \( \int_{12}^{B} (x + 13)^{-3/2} \, dx \) gives 99.9% of the area represented by the integral in (2A).

\[
99.9 \% \text{ of } 0.4 \text{ is } 0.3996
\]

So solve \( \frac{-2}{\sqrt{B+13}} + \frac{2}{5} = 0.3996 \)

\[
\text{to get } B = 24,999,987
\]