MATH 106A - CALCULUS II
WINTER 2007

QUIZ 6

NAME:

Show ALL your work CAREFULLY.

(a) Find the second-order Maclaurin polynomial $M_2(x)$ for the function $f(x) = \frac{1}{1+x^2}$.

Since $f(x) = \frac{1}{1+x^2}$, we have $f'(x) = -(1 + x^2)^{-2} \cdot 2x$ and $f''(x) = -[-2(1 + x^2)^{-3} \cdot 2x + (1 + x^2)^{-2} \cdot 2]$. It follows that $f(0) = 1$, $f'(0) = 0$, and $f''(0) = -2$. Thus,

$$M_2(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2} = 1 - x^2.$$  

(b) Find the third-order Taylor polynomial $P_3(x)$ for the function $g(x) = \sin x$ based at $x_0 = \pi/2$.

Note that $g'(x) = \cos x$, $g''(x) = -\sin x$, and $g'''(x) = -\cos x$. It follows that $g(\pi/2) = 1$, $g'(\pi/2) = 0$, $g''(\pi/2) = -1$, and $g'''(\pi/2) = 0$. Thus,

$$P_3(x) = 1 - \frac{(x - \pi/2)^2}{2}.$$