1. Find \( \int \frac{dx}{x^4 \sqrt{x^2 + 16}} \). Show ALL your work.

Hints: (1) Use an appropriate trig substitution.

(2) After making the substitution and simplifying, you may use the fact that \( \int \frac{\sec(t) \, dt}{(\tan(t))^4} = \frac{(\sin(t))^2 - (1/3)}{(\sin(t))^3} + C \)

\[
\begin{align*}
\text{We know } & \sin^2 t + \cos^2 t = 1 \\
\text{so } & \tan^2 t + 1 = \sec^2 t \quad \text{(by dividing } s^2 + c^2 = 1 \text{ by } \cos t) \\
\text{then } & 16 \tan^2 t + 16 = 16 \sec^2 t \\
\Rightarrow & \text{we have } x = 4 \tan t. \text{ So } x = 4 \tan t, \ dx = 4 \sec^2 t \, dt. \\
\text{Also (and the main reason we introduce the substitution)} & \\
\sqrt{x^2 + 16} = \sqrt{16 \tan^2 t + 16} = \sqrt{16 \sec^2 t} = 4 \sec t
\end{align*}
\]

Now, \( \int \) becomes \( \int \frac{4 \sec^2 t \, dt}{(4 \tan t)^4} = \int \frac{\sec t \, dt}{(4 \tan t)^4} \)

\[
= \int \frac{\sec t \, dt}{256 (\tan t)^4} = \frac{1}{256} \int \frac{\sec t \, dt}{(\tan t)^4} = \frac{1}{256} \left( \frac{(\sin t)^2 - (1/3)}{(\sin t)^3} \right) + C
\]

We must provide the answer to the original \( \int \) in terms of \( x \)’s. Since \( x = 4 \tan t \), we have \( x/4 = \tan t \), \( \text{opp \over \text{adj}} \Rightarrow \text{we should label the } \Delta \text{ as } \cdots \). The third side, \( \text{hyp} = \sqrt{x^2 + 16} = \sqrt{x^2 + 16} \).

Finally, since \( \sin t = \frac{\text{opp}}{\text{hyp}} \), we get \( \sin t = \frac{x}{\sqrt{x^2 + 16}} \) and this tells us the final answer is \( \frac{1}{256} \left( \frac{x^2}{(x/4)^4} \right) ^2 - (1/3) + C \).

2. Find \( \int \frac{4x - 8}{x^2 + 1} \, dx \). Show ALL your work.

Try a substitution: let \( u = x^2 + 1 \), \( du = 2x \, dx \).

\[ \begin{align*}
\text{He } \int \text{ becomes } & \int \frac{4x - 8}{x^2 + 1} \, dx = 2 \int \frac{2x - 4}{x^2 + 1} \, dx = 2 \left[ \frac{2x \, dx}{x^2 + 1} - 8 \int \frac{dx}{x^2 + 1} \right] \text{ just DO IT!}
\end{align*} \]

\[ \begin{align*}
&= 2 \ln |u| - 8 \arctan(x) + C \\
&= 2 \ln |x^2 + 1| - 8 \arctan(x) + C
\end{align*} \]

BONUS! Show why \( \int \frac{\sec(t) \, dt}{(\tan(t))^4} = \frac{(\sin(t))^2 - (1/3)}{(\sin(t))^3} + C \). Use the BACK of the quiz to answer this.

\[ \begin{align*}
\text{Use the BACK of the quiz to answer this.}
\end{align*} \]