SHOW ALL WORK, CLEARLY AND LEGIBLY, TO RECEIVE FULL CREDIT. CORRECT SPELLING, ORGANIZATION OF YOUR SOLUTION, AND PROPER USE OF MATHEMATICAL NOTATION ALL COUNT. YOU MAY USE A STAND-ALONE GRAPHING CALCULATOR, BUT NOT ANY INTERNET-BASED CALCULATORS. NO NOTES, BOOKS, OR OTHER ADDITIONAL RESOURCES ARE PERMITTED. GOOD LUCK!

1.) (10 pts.) The graphs below are $f$, $f'$, and $f''$. State which is which, and explain how you know this.

\[
\begin{align*}
\text{Dashed line: } & f \\
\text{Bold line: } & f' \\
\text{Thinner line: } & f''
\end{align*}
\]

$f$ increases throughout, so its slopes are always positive.

$f'$ is always positive.

$f'$ has slope 0 at about $x = 1.8$ and $x = 4.3$.

$f'' = 0$ at those $x$-values.$^1$

No curve has slope 0 at about $x = 1.3$, where the dashed line crosses the $x$-axis, so the dashed line cannot be the derivative of either other graph.$^1$
2.) (15 pts.)

a.) (5 pts.) Suppose \( \lim_{{x \to 5}} f(x) = 2 \) and \( \lim_{{x \to 5^+}} f(x) = 4 \). Is it possible that \( \lim_{{x \to 5^+}} f(x) = 3 \)? Justify your answer.

No. For \( \lim_{{x \to 5}} f(x) \) to exist at all, we would need \( \lim_{{x \to 5^-}} f(x) = \lim_{{x \to 5^+}} f(x) \), and this does not happen, so \( \lim_{{x \to 5}} f(x) = \text{D.N.E.} \)

b.) (5 pts.) Suppose \( g(x) = \frac{x^2 + 3x - 10}{x - 2} \). What is \( g(2) \)? [Note: \( g(x) \) is not related to \( f(x) \) in part (a).]

\[
g(2) = \frac{2^2 + 3 \cdot 2 - 10}{2 - 2} = \frac{0}{0} \rightarrow \text{D.N.E.}
\]

c.) (5 pts.) What is \( \lim_{{x \to 2}} \frac{x^2 + 3x - 10}{x - 2} \)?

\[
= \lim_{{x \to 2}} \frac{(x-2)(x+5)}{x-2} = \lim_{{x \to 2}} (x+5) = 7
\]
3.) (15 pts.)

a.) (5 pts.) Give an example of a polynomial, and describe in words what it means for a function to be a polynomial.

\[ f(x) = 7x^6 - 3x^4 - 2 \]

Real-valued coefficients, \( x \) raised to whole number exponents, terms added or subtracted together.

b.) (5 pts.) Give an example of a rational function, and describe in words what it means for a function to be a rational function.

\[ f(x) = \frac{2x^2 - 5}{9x^{10} + 3x} \]

Polynomial divided by a polynomial

c.) (5 pts.) Give an example of an exponential function, and describe in words what it means for a function to be an exponential function.

\[ y = 5^x \]

A constant base and a variable in the exponent
4.) (15 pts.) Shown below is a graph of $f'$ on its entire domain. The graph is NOT $f$. 

![Graph of $f'(x)$](image)

a.) (3 pts.) At which $x$-value(s) does $f$ have a stationary point?

Where $f' = 0$:

$x = 1, 3, 5$

b.) (3 pts.) At which $x$-value(s) does $f'$ have a stationary point?

At $x = 1, 2, 4, 5$

c.) (3 pts.) At which $x$-value(s) is $f$ greatest?

When $f'$ switches from positive to negative, so $f$ switches from increasing to decreasing at its local max:

$x = 3$

d.) (3 pts.) At which $x$-value(s) is $f$ increasing?

Where $f' > 0$: $x \in [0, 3]$

e.) (3 pts.) At which $x$-value(s) is $f$ concave up?

Where $f'$ is increasing:

$x \in [1, 2] \cup x \in [4, 5]$
5.) (15 pts.) For each of the following questions, let \( f(x) = \sqrt{x} + \frac{1}{x^3} \). On this page, you may complete the exercises using the Power Rule we learned for computing derivatives and antiderivatives.

\[
\int f(x) = x^{\frac{1}{2}} + x^{-3}
\]

a.) (5 pts.) Compute the general antiderivative \( F(x) \).

\[
F(x) = \frac{1}{\frac{1}{2}+1} x^{\frac{1}{2}+1} + \frac{1}{-3+1} x^{-3+1} + C
\]

\[
F(x) = \frac{2}{3} x^{\frac{3}{2}} - \frac{1}{2} x^{-2} + C
\]

b.) (5 pts.) Solve the initial value problem in which the differential equation is \( f(x) \) and the initial condition is \( F(1) = 3 \).

\[
F(1) = \frac{2}{3} (1)^{\frac{3}{2}} - \frac{1}{2} (1)^{-2} + C = 3
\]

\[
\frac{2}{3} - \frac{1}{2} + C = 3
\]

\[
\frac{1}{6} + C = 3
\]

\[
C = \frac{17}{6}
\]

\[
F(x) = \frac{2}{3} x^{\frac{3}{2}} - \frac{1}{2} x^{-2} + \frac{17}{6}
\]

c.) (5 pts.) Compute \( f'(x) \).

\[
f'(x) = \frac{1}{2} x^{\frac{-1}{2}} - 3 x^{-4}
\]
6.) (15 pts.) Consider the function \( f(x) = \ln(8x) \).

a.) (5 pts.) Draw \( f(x) \), showing the graph for \( x \)-values ranging from 0 to 5.

\[ \begin{array}{c}
\text{Graph of } f(x) = \ln(8x) \text{ with } x \text{-values from 0 to 5.}
\end{array} \]

b.) (5 pts.) Numerically zoom to estimate \( f'(2) \).

\[
\begin{align*}
f(2) &= 2.77 \\
f(2.1) &= 2.82 \\
f'(2) &\approx \frac{f(2.1) - f(2.0)}{2.1 - 2.0} = \frac{2.82 - 2.77}{0.1} = 0.5
\end{align*}
\]

c.) (5 pts.) Explain, referring to your graph, how the idea of numerical zooming leads us to the exact definition of the derivative at a point (such as at the point \( x = 2 \)).

\[
\begin{align*}
f(x) &= f(2) \\
f(x + h) \text{ is our } f(2.1) \text{ with } h = 0.1
\end{align*}
\]

The derivative at a point moves \( h \) infinitely close to 0 to compute the exact tangent lineslope. Otherwise we are approximating, using a secant line slope.
7.) (15 pts.) Use the limit definition of the derivative to compute $f'(x)$ for $f(x) = 3x^2 + 5x$. [NOTE: you may use the Power Rule to check your result, but that alone will earn you no credit.]

\[
\begin{align*}
    f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
    &= \lim_{h \to 0} \frac{[3(x+h)^2 + 5(x+h)] - [3x^2 + 5x]}{h} \\
    &= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 + 5x + 5h - 3x^2 - 5x}{h} \\
    &= \lim_{h \to 0} \frac{6xh + 3h^2 + 5h}{h} \\
    &= \lim_{h \to 0} \frac{h(6x + 3h + 5)}{h} \\
    &= \lim_{h \to 0} (6x + 3h + 5) \\
    &= 6x + 5
\end{align*}
\]