There are 6 total problems in this exam. On each problem, you must show all your work, or otherwise thoroughly explain your conclusions. There is always at least one step preceding a final answer. Units may be requested for your final answer; a point deduction will apply if they are omitted.

On the portion of the exam marked No Calculator, you will be allowed 30 minutes during which your calculator must be closed and put away. If you finish this section early, you may hand in your work early. However, only after you hand in the “no calculators” section will you be permitted to use a calculator. You may not return to the “no calculator” portion after handing it in.

Before beginning, ensure your calculator is set to Radians mode.

You will have 80 minutes to complete this exam.

<table>
<thead>
<tr>
<th>Question</th>
<th>Point Value</th>
<th>Your Score</th>
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<tbody>
<tr>
<td>No Calc.</td>
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<td><strong>Total</strong></td>
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</table>
Problem 1-NC. (25 points) Use the limit definition of derivative to compute \( f'(x) \) for the function

\[
f(x) = x e^x.
\]

**Hint:** Simplify using properties of exponentials. You will also need to know that \( \lim_{h \to 0} \frac{e^h - 1}{h} = 1 \).
**Problem 2-NC.** (25 points) In this question, you will determine enough properties of the function

\[ f(x) = ex + e^{-x} - e \]

to be able to graph it.

**Note:** remember, \( e \) is just a number. Its particular value, \( \approx 2.71828... \) is not important to this problem.

(a) (5 points) What is the vertical intercept of this function?

(b) (7 points) Determine on what interval(s) this function is increasing, and on what interval(s) it is decreasing.

(c) (5 points) Is this function always concave up? Always concave down? Neither? Why?

(d) (8 points) On the axes provided, sketch the graph of \( f \) using your answers to parts (a)—(c).
Problem 1. (25 points) This problem concerns the function
\[ g(t) = \frac{(t - 3)(t^2 - t + 1)}{t^2 - 4t + 3}. \]

(a) (8 points) Determine the domain of this function.

(b) (10 points) Using algebra, compute \( \lim_{t \to a} g(t) \) for each value of \( a \) not in the domain of \( g \). Explain what each result means about the continuity of \( g \).

(c) (7 points) At left is a partial graph of \( g(t) \). Fill in the gap, clearly indicating the nature of any discontinuities.
Problem 2. (25 points) A weight is attached to a spring and suspended in a container of motor oil. If it is allowed to oscillate, its vertical position (measured in cm above equilibrium) as a function of time $t$ in seconds might be given by the function

$$p(t) = 3e^{-t} \cos t.$$ 

(a) (10 points) Complete the data table below, and use your results to estimate the values of $p'(0.9)$, $p'(1)$, and $p'(1.1)$. Include units in your answers.

<table>
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<tr>
<th>$t$ (sec)</th>
<th>0.85</th>
<th>0.9</th>
<th>0.95</th>
<th>1</th>
<th>1.05</th>
<th>1.1</th>
<th>1.15</th>
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<tbody>
<tr>
<td>$p$ (cm)</td>
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$p'(0.9) \approx \quad p'(1) \approx \quad p'(1.1) \approx$

(b) (10 points) Use your answers to part (a) to estimate $p''(1)$, with units. What does this answer mean in practical terms?

(c) (5 points) Is it reasonable, based on your answers, to expect that $p(t)$ satisfies the differential equation

$$p'' + 2p' = -2p \quad ?$$

Why or why not?
**Problem 3.** (25 points) Shown at left is a graph of the derivative of a function $h$. Use the graph to answer the following questions.

(a) (6 points) On what interval(s) is $h(x)$ decreasing?

(b) (6 points) List the $x$ values of all local minimum and local maximum point(s) of $h$, and justify your answers in one sentence.

(c) (6 points) The graph of $h'$ is concave up on the interval $(1.5, 3.0)$. What does this mean about the graph of $h(x)$ on that interval?

(d) (7 points) Using the axes provided at left, sketch a possible graph of the function $h(x)$. The vertical scale is not important, only the shape and horizontal location.
**Problem 4.** (25 points) The latest press booklet for the 2009 Lotus Exige S-240 sports car claims it can accelerate from 0—60 mph in 4.0 seconds flat. A recent test-track run showed that under full throttle, the velocity of the car is modeled by the function

\[ v(t) = 40 \sqrt{t} - 5t, \]

where \( v \) is measured in mph and \( t \) in seconds.

(a) (15 points) According to this model, after how many seconds will the car reach its maximum velocity, and what is the maximum velocity?

**Note:** do this symbolically, showing your work. You may include a graph or data table if you wish, but your answer must be exact.

(b) (10 points) Determine the car’s distance function \( d(t) \) — an antiderivative of its velocity — and use it to find the distance the car traveled during the first 10 seconds of this time trial.

**Note:** write out the units of the antiderivative in your answer. Convert them if you wish.