NAME:

Show ALL your work CAREFULLY.

(a) Use the technique of trigonometric substitution to evaluate the following indefinite integral. [Recall that \( \sec^2 \theta = 1 + \tan^2 \theta \).]

\[
\int \frac{dx}{x^2 \sqrt{4 - x^2}}
\]

Let \( x = 2 \sin \theta \) so that \( dx = 2 \cos \theta d\theta \). It follows that

\[
\int \frac{dx}{x^2 \sqrt{4 - x^2}} = \int \frac{2 \cos \theta \ d\theta}{(2 \sin \theta)^2 \sqrt{4 - 4 \sin^2 \theta}} = \frac{1}{4} \int \frac{d\theta}{\sin^2 \theta} = \frac{1}{4} \int \csc^2 \theta \ d\theta = -\frac{1}{4} \cot \theta + C \quad \text{(using a right angle triangle for the substitution \( \sin \theta = \frac{x}{2} \))}
\]

\[
= -\frac{\sqrt{4 - x^2}}{4x} + C.
\]

(b) Evaluate the following indefinite integral. Be sure to indicate what method of integration you use.

\[
\int \frac{x \ dx}{\sqrt{4 - x^2}}
\]

Let \( w = 4 - x^2 \) so that \( dw = -2x \ dx \) or \( x \ dx = \frac{dw}{2} \). It follows that

\[
\int \frac{x \ dx}{\sqrt{4 - x^2}} = \int -\frac{dw}{2\sqrt{w}} = -\sqrt{w} + C = -\sqrt{4 - x^2} + C.
\]

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