1. Let \( f(x) = 3 + \sqrt{x + 5} \).

(a) What is the natural domain of \( f \)? \([-5, \infty)\), which means all reals greater than or equal to 5

(b) What is the range of \( f \)? \([3, \infty)\), which means all reals greater than or equal to 3

2. For the graph of \( f \) shown, answer the following.

(a) Evaluate the following.
   i. \( f'(2) = 0 \)
   ii. \( f(3) = 2 \)
   iii. \( \lim_{x \to 3^-} f(x) = -2 \)
   iv. \( \lim_{x \to 3^+} f(x) = 3 \)
   v. \( \lim_{x \to 3} f(x) \) does not exist
   vi. \( \lim_{x \to -2} f(x) = 0 \)

(b) Where is \( f \) discontinuous? at \( x = 3 \)

(c) Where does \( f' \) fail to exist?
   at \( x = -3, -1, 0, 3 \)

3. Let \( f(x) = 3x^2 - 2x \).

   (a) Compute the average rate of change of \( f \) on the interval \([2, 2.1]\).
   \[
   \frac{f(2.1) - f(2)}{2.1 - 2} = \frac{9.03 - 8}{0.1} = 10.3
   \]

   (b) Using the limit definition of the derivative, find \( f'(x) \).
   \[
   f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \quad \text{provided this limit exists}
   = \lim_{h \to 0} \frac{3(x + h)^2 - 2(x + h) - (3x^2 - 2x)}{h}
   = \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 2x - 2h - 3x^2 + 2x}{h}
   = \lim_{h \to 0} \frac{h(6x + 3h - 2)}{h}
   = \lim_{h \to 0} (6x + 3h - 2)
   = 6x - 2
   \]

   (c) Find the equation of the tangent line to \( f \) at \( x = 2 \).
   We want \( y = mx + b \). \( m = f'(2) = 6 \cdot 2 - 2 = 10 \), so \( y = 10x + b \).
   When \( x = 2 \), \( y = f(2) = 3 \cdot 2^2 - 2 \cdot 2 = 8 \).
   Thus, \( 8 = 10 \cdot 2 + b \), so \( b = -12 \) and we have \( y = 10x - 12 \).

   (d) How would the derivative of \( g(x) = f(x) + 5 \) compare to \( f'(x) \)?
   The graph of \( y = f(x) + 5 \) is the graph of \( y = f(x) \) shifted vertically by 5 units, but this has no effect on the slope of the graph, so \( g'(x) = f'(x) \).
(e) **How would the derivative of** \( h(x) = 5f(x) \) **compare to** \( f'(x) \)?

The graph of \( y = 5f(x) \) is the graph of \( y = f(x) \) stretched vertically by a factor of 5; this also results in slopes that are 5 times greater at any given \( x \)-value. Thus, \( h'(x) = 5f'(x) \).

Note that we get the same result by considering our derivative rule \( \frac{d}{dx}[kf(x)] = kf'(x) \) where \( k = 5 \).

4. **Given the graph of** \( f \), **sketch a graph of** \( f' \) **and a graph of** \( F \), **an antiderivative of** \( f \) **such that** \( F(0) = -2 \).

![Graph of f(x)](image1)

![Graph of f'(x)](image2)

Note: The concave up portion in the middle of the graph of \( f \) is a perfect parabola, so its derivative \( (f') \) is linear; since you don’t know the equation for \( f \), your graph of \( f' \) may be concave up/down there.
5. Shown below is a graph of $f'$ on its entire domain. The graph is NOT $f$.

At which $x$-value(s)

(a) does $f$ have a stationary point? $c, e, m$
(b) does $f$ have a local max? $e$
(c) does $f$ have a local min? $c$
(d) does $f'$ have a stationary point? $d, i, m$
(e) does $f'$ have a local max? $d, m$
(f) does $f'$ have a local min? $i$
(g) is $f$ greatest? $a$
(h) is $f$ least? $n$
(i) is $f'$ greatest? $d$
(j) is $f'$ least? $a$
(k) is $f''$ greatest? $b$
(l) is $f''$ least? $g$

On what interval(s) * is

(a) $f$ increasing? $c$ to $e$
(b) $f$ decreasing? $a$ to $c$ and $e$ to $n$

(c) $f'$ increasing? $a$ to $d$ and $i$ to $m$
(d) $f'$ decreasing? $d$ to $i$ and $m$ to $n$
(e) $f$ concave up? $a$ to $d$ and $i$ to $m$
(f) $f$ concave down? $d$ to $i$ and $m$ to $n$

* Whether to include the endpoints of these intervals will depend on your instructor’s definitions.

6. Suppose that $T(t)$ gives the temperature in Lewiston as a function of time. In each of the following situations, determine if the signs of $T$, $T'$, and $T''$ are positive, negative, zero, or unknown.

(a) The temperature is 60 degrees and falling steadily.
   The temperature is 60, so we know $T$ is positive.
   The temperature is falling, so we know $T'$ is negative.
   The temperature is falling steadily, so we know the graph is linear, and $T''$ is zero.

(b) The temperature is rising more and more slowly.
   We don’t know whether the temperature is above or below zero, so the sign of $T$ is unknown.
   The temperature is rising, so we know $T'$ is positive.
   The temperature is rising more and more slowly, so we know the graph of $T$ is concave down, and $T''$ is negative.

(c) The temperature is $-5$ degrees and rising.
   The temperature is $-5$, so we know $T$ is negative.
   The temperature is rising, so we know $T'$ is positive.
   We don’t know the concavity of the graph of $T$, so the sign of $T''$ is unknown.

7. The table below gives some values for a function $f(x)$ whose derivative exists at all $x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
<th>1.1</th>
<th>1.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>5.0</td>
<td>6.2</td>
<td>7.3</td>
<td>8.2</td>
<td>9.0</td>
</tr>
</tbody>
</table>
(a) Solve the IVP (initial value problem)
\[ \frac{f(1.1) - f(1.0)}{1.1 - 1.0} = \frac{8.2 - 7.3}{1.1 - 1.0} = 9 \]

(b) Find the derivatives of the following.

Based on the data, is \( f''(1.0) \) positive or negative?

Using the same procedure as in the previous part, we can make the following estimates.

\[ f'(0.85) \approx 12 \quad f'(0.95) \approx 11 \quad f'(1.05) \approx 9 \quad f'(1.15) \approx 8 \]

We see from these estimates that \( f'(x) \) appears to be decreasing near \( x = 1 \). If \( f'(x) \) is decreasing, then \( f''(x) \) is negative (that is, \( f(x) \) is concave down).

8. Find the derivatives of the following.

(a) \( y = 2 + 3x + x^4 + 5x^6 \)
\[ y' = 3 + 4x^3 + 30x^5 \]

(b) \( y = \sqrt[3]{x} + \frac{1}{x^6} + \frac{x}{6} + \frac{\pi}{x} + 6^{1/2} \)

First, rewrite \( y \) to make it easier to apply our derivative rules:
\[ y = x^{1/6} + x^{-6} + \frac{1}{6}x + 6x^{-1} + \frac{\pi}{6} + 6^{1/2} \]
\[ y' = \frac{1}{6}x^{-5/6} + (-6)x^{-7} + \frac{1}{6} + (6)(-1)(x^{-2}) + 0 + 0 \]

If necessary, we can rewrite this using exponent rules: \( x^{-n} = 1/x^n \) and \( x^{m/n} = \sqrt[n]{x^m} \).
\[ y' = \frac{\sqrt[3]{x}}{6x} \cdot \left( \frac{1}{x^3} + \frac{1}{6} - \frac{6}{x^2} \right) \]

9. Find antiderivatives of the following.

(a) \( y = \pi + 3x^2 \)
antiderivative = \( \pi x + x^3 + C \)

(b) \( y = 4x^5 - \frac{1}{x^6} = 4x^5 - x^{-6} \)
antiderivative = \( \frac{4x^6}{6} - \frac{x^{-5}}{-5} + C = \frac{2x^6}{3} + \frac{1}{5x^5} + C \)

10. Solve the IVP (initial value problem) \( 1 = x^3 - y'(x) \) if \( y(2) = 13 \).

We begin by isolating \( y'(x) \). This gives \( y'(x) = 1 - x^3 \)

Next we find the antiderivative of \( y'(x) \): \( y(x) = -x + \frac{x^4}{4} + C \).

Now we plug in the 2 and the 13 to find the value of \( C \).
\[ 13 = -2 + \frac{4}{4} + C \]
\[ 13 = -2 + 4 + C \]
\[ 11 = C \]

So, the solution to this IVP is \( y(x) = x - \frac{x^4}{4} + 11 \).