YOUR GRADE IS BASED ON CORRECTNESS, COMPLETENESS, AND CLARITY ON EACH EXERCISE. EXPLAIN ALL ANSWERS COMPLETELY. YOU MAY USE A CALCULATOR, BUT NO NOTES, BOOKS, OR OTHER STUDENTS. GOOD LUCK!

1.) (10 pts.) Check whether \( y = \ln x + x^2 + e^x \) solves the differential equation \( y'' + xy' = 3 \).

\[
y' = \frac{1}{x} + 2x + e^x
\]
\[
y'' = -\frac{1}{x^2} + 2 + e^x
\]

Check:

\[
\left( -\frac{1}{x^2} + 2 + e^x \right) + x \left( \frac{1}{x} + 2x + e^x \right) = 3
\]

\[-\frac{1}{x^2} + 2 + e^x + 1 + 2x^2 + xe^x \neq 3
\]

\[-\frac{1}{x^2} + e^x + 2x^2 + xe^x \neq 3 \text{ NO!}
\]

So: not a solution.
2.) (15 pts.) Suppose that \( g(x) = f(x) + 3 \) and that \( f'(x) \) exists for all \( x \).

a.) Explain how the graphs of \( f \) and \( g \) are related.

The graph of \( g \) looks like the graph of \( f \), but is shifted up 3 units.

b.) How is the graph of \( g' \) related to the graph of \( f'' \)? Explain.

\[
g'(x) = f'(x) \quad \text{These are identical.}
\]

(This makes sense because \( g \) is a vertical shift of \( f \), but the slopes at any \( x \)-value are the same.)

c.) If \( f'(1) = 5 \), what is \( g'(1) \)?

\[
g'(1) = f'(1) \quad \text{(based on part (i))}
\]

so \( g'(1) = 5 \).
3. (15 pts.) Use the limit definition of the derivative to compute \( f'(x) \) if \( f(x) = -3x^2 + 5x \). Be sure to write out all appropriate notation at each step. (Note: using the Power Rule only earns you no credit on this problem, though it is OK to use the Power Rule to check your answer.)

\[
\begin{align*}
    f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
    &= \lim_{h \to 0} \frac{-3(x+h)^2 + 5(x+h) - (-3x^2 + 5x)}{h} \\
    &= \lim_{h \to 0} \frac{-3x^2 - 6xh - 3h^2 + 5x + 5h + 3x^2 - 5x}{h} \\
    &= \lim_{h \to 0} \frac{h(-6x - 3h + 5)}{h} \\
    &= \lim_{h \to 0} (-6x - 3h + 5) \\
    &= -6x + 5
\end{align*}
\]
4.) (15 pts.) The graph below shows the function $f$. Use it to answer the questions about an antiderivative function $F$, and about $f$.

![Graph of function $f(x)$]

a.) At which $x$-value(s) does $F$ have a local minimum?

Where: $F'$ (which is $f$) has a zero and is moving from negative to positive:

$x \approx 0.7, 6.4$

b.) On which interval(s) is $F$ increasing?

Where $f$ is positive:

$(0.7, 2.9) \cup (6.4, 8.8)$

c.) On which interval(s) is $F$ concave down?

Where $f$ is decreasing:

$(1.8, 3.9) \cup (5, 5.6) \cup (7.7, 10)$

d.) At which $x$-value(s) does $F$ have an inflection point?

Where $f$ changes from increasing to decreasing, or decreasing to increasing:

$x \approx 1.8, 3.9, 5, 5.6, 7.7$

e.) On which interval(s) is $f$ decreasing?

$(1.8, 3.9) \cup (5, 5.6) \cup (7.7, 10)$
5.) (15 pts.) The graph below shows the function $f$. Use it to answer the questions below.

![Graph of f(x)](image)

a.) Use the grid to estimate the values of $f'(1)$, $f'(5)$, and $f''(8)$.

- $f'(1) \approx 3$
- $f'(5) \approx 0$
- $f''(8) \approx \frac{-3}{2}$

b.) On which interval(s) is $f'' > 0$?

Where $f$ is concave up:

$$(3, 4.5) \cup (5.2, 6.7)$$

c.) Using the axes below, sketch a graph of $f'$.
6.) (15 pts.) For each of the following graphs, answer the questions below and justify your responses.

1.) Is this the graph of a function?
   Yes: there is just one output for each input.

2.) Does the graph have even symmetry, odd symmetry, or neither?
   Even: mirror image about y-axis.

1.) Is this the graph of a function?
   No: vertical line test (above) shows two outputs for a single input.

2.) Does the graph have even symmetry, odd symmetry, or neither?
   Neither: it is not a mirror image about the y-axis (even) nor can we rotate 180° about the origin and have the same shape (odd).
7.) (15 pts.) Use the graph of $f(x)$ below to answer the questions about limits.

![Graph of f(x)](image)

a.) What is $\lim_{x \to 2} f(x)$? 0

b.) What is $\lim_{x \to 2} f'(x)$? \text{DNE}

c.) What is $\lim_{x \to 0^+} f(x)$? 4

d.) What is $\lim_{x \to 0} f(x)$? \text{DNE}

e.) What is $\lim_{x \to 6^-} f(x)$? 0

**BONUS:** Write a poem about Calculus. You may use the back of this page or attach a page you have brought with you. (If you are attaching a page, please make sure your name is on it.)