Correct answers accompanied by incorrect or incomplete work will not receive full credit.

1. Consider the triangle with vertices \( A = (1, 1, 1) \), \( B = (3, -2, 3) \), and \( C = (3, 4, 6) \). (Figure may not be drawn to scale.)

(a) (10 points) Find \( \angle ABC \), i.e., find \( \theta \). In your work use correct vector notation.

\[
\mathbf{v} = \mathbf{BC} = (3, 4, 6) - (3, -2, 3) = (0, 6, 3)
\]

\[||\mathbf{v}|| = \sqrt{36 + 9} = \sqrt{45}\]

\[
\mathbf{v} = \mathbf{BA} = (1, 1, 1) - (3, -2, 3) = (-2, 3, -2)
\]

\[||\mathbf{v}|| = \sqrt{4 + 9 + 4} = \sqrt{17}\]

\[\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{||\mathbf{u}|| ||\mathbf{v}||} = \frac{9 + 18 + 6}{6 \cdot \sqrt{17}} \approx 0.4339\]

\[\theta \approx \cos^{-1}(0.4339) = 1.12 \text{ rad} \approx 64.30\]°

(b) (10 points) Find the area of the triangle. In your work use correct vector notation.

\[\text{Area} = \frac{1}{2} ||\mathbf{v} \times \mathbf{w}||\]

\[
\mathbf{v} \times \mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & -2 \\ -2 & 3 & -2 \\ 0 & 6 & 3 \end{vmatrix} = (9 - 12)\mathbf{i} - (-6 - 0)\mathbf{j} + (12 - 0)\mathbf{k} = 3\mathbf{i} + 6\mathbf{j} - 12\mathbf{k}
\]

\[||\mathbf{v} \times \mathbf{w}|| = \sqrt{21^2 + 36 + 144} = \sqrt{621}\]

\[\text{Area} = \frac{1}{2} \sqrt{621} \approx 12.46\]
2. Consider the equation $Ax^2 + By^2 + Cz^2 = D$. Fill in each blank with a single number that makes the statement correct.

(a) (10 points) If $A = \frac{1}{2}$, $B = \frac{-1}{2}$, $C = \frac{-1}{2}$, and $D = \frac{1}{2}$ then the graph of the equation is a hyperboloid of 2 sheets with axis being the $x$-axis.

($\Rightarrow$ 2 of $A$, $B$, $C$ must be neg and $D > 0$

pos coeff corresponds to axis so $A > 0$

(other answers are possible)

(b) (10 points) If $A = \frac{1}{2}$, $B = \frac{-1}{2}$, $C = \frac{1}{2}$, and $D = \frac{0}{2}$ then the graph of the equation is a (double) cone with axis being the $y$-axis.

($\Rightarrow$ $y^2 = x^2 + z^2$ is such a cone.

$\Rightarrow$ $x^2 - y^2 + z^2 = 0$

(other answers are possible)

3. (10 points) Write the equation of the ellipsoid with center $(2, -3, 5)$ that is tangent to the planes $x = 0$, $y = 0$, and $z = 0$.

$x$-semi axis = 2 blc ctr is 2 units from $x=0$ plane

$y$-semi axis = 3 blc ctr is 3 units from $y=0$ plane

$z$-semi axis = 5 blc ctr is 5 units from $z=0$ plane

So eqn is

$$\frac{(x-2)^2}{2^2} + \frac{(y+3)^2}{3^2} + \frac{(z-8)^2}{5^2} = 1.$$
4. (a) (15 points) Sketch the $z = 0$, $x = 0$, and $y = 0$ traces of $f(x, y) = |x + y|$.

\[ z = 0; \quad 0 = |x + y| \]
\[ 0 = x + y \]
\[ -x = y \]

\[ x = 0; \quad z = |y| \]

\[ y = 0; \quad z = |x| \]

(b) (5 points) Which of the following graphs is the graph of $f(x, y) = |x + y|$?

**Not II** b/c it has the wrong $z = 0$ trace

Bottom of "box" intersects graph

**Not III** b/c it has wrong $x = 0$ & $y = 0$ traces.

Or b/c it has negative $z$-values.

(Also it's clearly a plane and $f(x, y) = (x + y)$ isn't.)

It also has $y = 0$ trace

and $x = 0$ trace

which are wrong.
5. Consider the lines $\vec{l}_1(t) = (6t + 1, 3t - 1, 2t + 2)$ and $\vec{l}_2(t) = (t + 3, \frac{1}{2}t + 1, \frac{1}{3}t - 1)$.

(a) (10 points) Show that $\vec{l}_1(t)$ and $\vec{l}_2(t)$ are parallel.

$$\vec{m}_1 = (0, 3, 2), \quad \vec{m}_2 = (1, \frac{1}{2}, \frac{1}{3})$$

$$\vec{n} = 6\vec{m}_2$$ so the vectors are parallel hence the lines are.

(b) (10 points) Write the equation of the plane through these two lines. Your final answer must have the form $Ax + By + Cz = D$.

We need a point $\vec{x}_0$ on plane and $\vec{n} \perp$ to plane.

Since $\vec{l}_1(t)$ is on the plane, we can take $\vec{x}_0 = (1, -1, 2)$ similarly using $\vec{l}_2(t)$ we could take $(3, 1, -1)$ to be $\vec{x}_0$.

Since $\vec{m}_1$ and $\vec{m}_2$ are linearly dependent, we can't cross them to find $\vec{n}$. Instead take $(1, -1, 2) - (3, 1, -1) = \vec{v} = (-2, -2, 3)$

and then $\vec{n} = \vec{v} \times \vec{m}_1$, or use $\vec{n} = \vec{v} \times \vec{m}_2$

$\vec{n} = \vec{v} \times \vec{m}_1 = (-13, 32, 6)$

Plane: $(\vec{x} - \vec{x}_0) \cdot \vec{n} = 0 \rightarrow -13x + 32y + 6z = -23$

6. (10 points) Write the parametrization for the portion of the ellipse pictured. Where the marked points correspond to the indicated values of $t$. If we didn't have to worry about orientation, the parametrization would be

$$\vec{r}(t) = (4\cos t - \frac{3}{2}, 2\sin t + 1)$$

to make orientation counterclockwise

$$\vec{r}(t) = (4\cos t - \frac{3}{2}, -2\sin t + 1)$$

$0 \leq t \leq \frac{3\pi}{2}$. 

**Center**

**y-semi = 2**

**x-semi axis**

**y = x-semi axis**

**$t = \pi/2$**

**$t = 0$**