Correct answers accompanied by incorrect or incomplete work will not receive full credit.

1. (4 points each) The graph of $f(x)$ is given. Evaluate the following (assume the tickmarks occur at 1, 2, etc).

   \[ y \]
   \[ x \]

(a) \( \lim_{{x \to 0^-}} f(x) = 1 \cdot 5 \)

(b) \( \lim_{{x \to 0^+}} f(x) = 4 \)

(c) \( \lim_{{x \to 0}} f(x) \) does not exist

(d) \( f(0) = 4 \)

(e) \( \lim_{{x \to -2}} f(x) \) does not exist

(f) \( f(-2) = 1 \cdot 5 \)
2. (6 points) The graph below is a graph of \( f(x) \). Estimate \( f'(-3) \).

\[ f'(3) \approx \frac{5-0}{-2-(-3)} = 5 \]

3. (6 points) Let \( g(x) = \tan x \). Use secant line(s) to numerically estimate \( g'(\pi/3) \). (Make sure your calculator is in RADIANT mode.)

\[ \pi/3 \approx 1.0472 \]
\[ g'(\pi/3) \approx 1.7321 \]
\[ g(1) \approx 1.5574 \]

4. (6 points) Let \( h(x) = 3x^2 + \frac{6}{x^3} - 4\sqrt{x^3} + 7x^{-2} + 14 \). Calculate \( h'(4) \).

\[ h'(x) = 6x - 18x^{-4} - 6x^{-1/2} - 14x^{-3} \]
\[ h'(4) = 6(4) - 18(4^{-4}) - 6(4^{-1/2}) - 14(4^{-3}) \approx 11,710.9 \]

5. (4 points) Let \( U(t) \) be the number of people unemployed in a country \( t \) months after the election of a new president. What does the statement \( U'(20) = -10,000 \) mean in this context? Include units in your answer.

The number of people unemployed 20 months after the election is decreasing at a rate of 10,000 people per month.
6. (5 points each) The graph below is a graph of $f(x)$.

Let $F(x)$ be an antiderivative of $f(x)$. 

(a) For what value(s) of $x$ (if any) does $F$ have a local maximum? Explain your answer.

Stationary points at $F'(x) = f(x) = 0$ so $x = x_3$, $x = x_8$ are stationary points.

So by the first derivative test $F$ has no local max.

(b) For what value(s) of $x$ (if any) does $F$ have a local minimum? Explain your answer.

As reasoned in (a) $F$ has a local min at $x = x_8$.

(c) For what value(s) of $x$ (if any) does $F$ have an inflection point? Explain your answer.

$F(x)$ has inflection points when $F'(x) = f(x)$ has local extrema. So $F$ has inflection points at $x = x_3$ and $x = x_6$. 
7. (5 points each) Suppose $g'(w) = \sqrt{w} - 3$. JUSTIFY your answer to each of the following questions WITHOUT GRAPHING $g'$.

(a) What is the natural domain of $g'(w)$?

$[0, \infty)$ b/c we can't take the square root of a negative number.

(b) Is $-3$ in the range of $g'(w)$?

Yes b/c $g'(0) = \sqrt{0} - 3 = -3$

So it is a possible output.

(c) Is $g$ concave up at $w = 4$?

$g'' = \frac{1}{2} w^{-1/2}$

$g''(4) = \frac{1}{2} (4)^{-1/2} = \frac{1}{4}$

which is positive. So $g$ is concave up.

(d) $g$ has a stationary point at $w = 9$. On $g$, is $w = 9$ a local maximum, local minimum, or neither?

We'll use the second derivative test.

From above $g''(w) = \frac{1}{2 \sqrt{w}}$. So $g''(9) = \frac{1}{2 \sqrt{9}} = \frac{1}{6}$

So $g$ is concave up at $w = 9$, so $g$ has a local min.

8. (5 points) Let $h(x) = 3x^2 + \frac{6}{x^3} - 4\sqrt{x^3} + 7x^{-2} + 14$. Find an antiderivative of $h$. $\alpha^+ \quad w = 9$.

$h(x) = 3x^2 + \frac{6}{x^3} - 4x^{3/2} + 7x^{-2} + 14$

And $H(x) = H(x) = x^3 + \frac{6x^{-2}}{-2} - \frac{4x^{5/2}}{5/2} + \frac{7x^{-1}}{-1} + 14x + C$

$H(x) = x^3 - 3x^{-2} - \frac{6}{5}x^{5/2} - 7x^{-1} + 14x + C$
9. (6 points) Sketch the graph of a continuous function \( g(x) \) over the interval \([-1, 5]\) that has the following properties:

- \( g(2) = 1 \)
- \( g'(x) > 0 \) on the interval \([-1, 3)\), \( g'(3) \) does not exist, and \( g'(x) > 0 \) on the interval \((3, 5]\). Increasing
- \( g''(x) < 0 \) on the interval \([-1, 3)\), \( g''(x) = 0 \) on the interval \([3, 5]\).
  Concave down
  \[ y = \text{line on } [3, 5] \]

10. (5 points) Let \( f(x) = \frac{3}{x+2} \). Fill in all of the empty spaces in following equation.

One possible answer is below

\[
f'(7) = \lim_{h \to 0} \frac{f(7+h) - f(7)}{h} = \lim_{h \to 0} \frac{\left( \frac{3}{7+h+2} \right) - \left( \frac{3}{7+2} \right)}{h} \quad (f \text{ should NOT appear in this line})
\]

11. (3 points) How many inches of snow do you think Lewiston will get this weekend?