NAME:

Show ALL your work CAREFULLY.

(a) Use the technique of integration by parts to find the following definite integral

\[ \int_0^1 x e^{-x} \, dx. \]

Let \( u = x \) and \( dv = e^{-x} \, dx \). Then \( du = dx \) and \( v = -e^{-x} \). Thus,

\[ \int_0^1 x e^{-x} \, dx \stackrel{IBP}{=} x(-e^{-x}) \bigg|_0^1 - \int_0^1 -e^{-x} \, dx \]
\[ = x(-e^{-x}) - e^{-x} \bigg|_0^1 \]
\[ = [(1)(-e^{-1}) - e^{-1}] - [0 - e^0] = 1 - \frac{2}{e}. \]

(b) Use the technique of partial fractions to evaluate the following indefinite integral.

\[ \int \frac{2x + 1}{x^2 - 7x + 12} \, dx \]

Write

\[ \frac{2x + 1}{x^2 - 7x + 12} = \frac{2x + 1}{(x - 4)(x - 3)} = \frac{A}{x - 4} + \frac{B}{x - 3}. \]

It follows that

\[ 2x + 1 \equiv A(x - 3) + B(x - 4) = (A + B)x + (-3A - 4B). \] (this is the numerator)

By equating the coefficients of the constant and the \( x \) terms, we have \( 2 = A + B \) and \( 1 = -3A - 4B \). Solving for \( A \) and \( B \), we obtain \( A = 9 \) and \( B = -7 \).

Now,

\[ \int \frac{2x + 1}{x^2 - 7x + 12} \, dx = \int \frac{9}{x - 4} - \frac{7}{x - 3} \, dx = 9 \ln |x - 4| - 7 \ln |x - 3| + C. \]