1. 3 pts each Let \( g(t) = (t + 1)(t - 2)^2 \).
   
   a. Is \( g \) even, or odd, or neither? EXPLAIN.

   b. List all the roots of \( g \).

   c. Is \( g \) increasing, decreasing, or neither over the interval \([0,2]\)? Justify your answer using calculus.

   d. What is the concavity of \( g \) over the interval \([-2,1]\)? Justify your answer using calculus.

2. 5 pts Sketch a graph of the Price Index as a function of time that is consistent with the scenario described in the following sentence: “The cost of living has been on the rise for the past 10 years. The good news is that prices have been rising at a decreasing rate.” (Let \( t = 0 \) correspond to 10 years ago.)
3. **2 pts each** Suppose an object is moving horizontally towards and away from some starting point (the origin). Sketch the object’s position vs. time graph

a. if the object is standing still.

b. if the object is moving *away* from the origin at a constant velocity.

c. if the object is moving *towards* the origin at a constant velocity.

d. if the object is moving *away* from the origin at a steadily *increasing* velocity.

e. if the object is moving *towards* the origin at a steadily *decreasing* velocity.
4. **4 pts each** Suppose that hot coffee is poured into a cup in a 70 degree Fahrenheit room. Let \( T(t) \) represent the temperature (in degrees Fahrenheit) of the coffee in the cup at time \( t \) minutes after it was poured.

a. What units are associated with \( T'(t) \)?

b. Is \( T(5) > 0 \)? Explain your answer (use common sense).

c. Is \( T'(5) > 0 \)? Explain your answer (use common sense).

5. **4 pts each** Suppose that \( f(1) = 1 \) and \( f'(x) = \sqrt{x^3} \) for \( x \geq 0 \). [NOTE: I have given the formula for \( f'(x) \) not \( f(x) \).]

a. Find an equation for the line tangent to \( f \) at \( x = 1 \).

b. *Using the tangent line*, estimate \( f(1.1) \).

c. Is your estimate too big or too small? *Using calculus*, justify your answer. (Hint: Is the tangent line above or below \( f \)? How can you figure this out?)
6. **5 pts each** Let $f$ be a function such that

$$
\lim_{h \to 0} \frac{f(3 + h) - f(3)}{h} = -2.
$$

**a.** Is it possible that

$$
\lim_{x \to 3} \frac{f(x) - f(3)}{x - 3} = 4?
$$

Justify your answer.

**b.** If $f(5) = 12$ and $f(3) = 2$, what is the average rate of change of $f$ on $[3, 5]$?

7. **3 pts each** Suppose that $f$ is continuous at $x = 3$ and that

$$
\lim_{x \to 3} f(x) = 17.
$$

Indicate whether each statement about $f$ MUST be true, MIGHT be true, or CANNOT be true. Justify your answers.

**a.** $3$ is in the domain of $f$.

**b.** $f(3) = 17$.

**c.**

$$
\lim_{x \to 3} f(x) = 17.
$$
8. **4 pts each** For each of the functions below, find $f'(x)$.

   a. $f(x) = \sqrt{x^3} + 4$

   b. $f(x) = 4/\sqrt{x} + \pi\sqrt[3]{x}$

   c. $f(x) = 4e^x - 3/\sqrt[3]{x^3}$

9. **5 pts each** A gardener wants to enclose 1000 square meters of land using as little fencing as possible. The garden is to be rectangular; one side, unfenced, lies along a river.

   a. Let $x$ be the two sides of the garden perpendicular to the river. Find an expression for $L(x)$ the total length of the fence, in terms of $x$.

   b. *Using calculus*, find the minimum amount of fencing needed. Show your work.
10. A diver’s height above the ground is given by $h(t) = t^2 - 7t + 12$ feet on the interval $[0, 4]$ (with $t$ measured in seconds).

a. **2 pts** What is the diver’s velocity at time $t$?

b. **2 pts** What is the lowest depth the diver reaches (on the interval given)?

c. **2 pts** What is the highest point of the diver above the ground (on the interval given)?

d. **2 pts** What is the diver’s velocity at her highest point?