1. Let \( f(x) = 3 + \sqrt{x + 5} \).

   (a) **What is the natural domain of \( f \)?** \([-5, \infty)\), which means all reals greater than or equal to 5

   (b) **What is the range of \( f \)?** \([3, \infty)\), which means all reals greater than or equal to 3

2. For the graph of \( f \) shown, answer the following.

   (a) Evaluate the following.

      i. \( f'(-2) = 0 \)
      ii. \( f(3) = 2 \)
      iii. \( \lim_{x \to 3^-} f(x) = -2 \)
      iv. \( \lim_{x \to 3^+} f(x) = 3 \)
      v. \( \lim_{x \to 3} f(x) \) does not exist
      vi. \( \lim_{x \to 2} f(x) = 0 \)

   (b) **Where is \( f \) discontinuous?** at \( x = 3 \)

   (c) **Where does \( f' \) fail to exist?**

      at \( x = -3, -1, 0, 3 \)

3. Let \( f(x) = 3x^2 - 2x \).

   (a) Compute the average rate of change of \( f \) on the interval \([2, 2.1]\).

      \[
      \frac{f(2.1) - f(2)}{2.1 - 2} = \frac{9.03 - 8}{0.1} = 10.3
      \]

   (b) Using the limit definition of the derivative, find \( f'(x) \).

      \[
      f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
      \]

      \[
      = \lim_{h \to 0} \frac{3(x + h)^2 - 2(x + h) - (3x^2 - 2x)}{h}
      \]

      \[
      = \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 2x - 2h - 3x^2 + 2x}{h}
      \]

      \[
      = \lim_{h \to 0} \frac{h(6x + 3h - 2)}{h}
      \]

      \[
      = \lim_{h \to 0} (6x + 3h - 2)
      \]

      \[
      = 6x - 2
      \]

   (c) **Find the equation of the tangent line to \( f \) at \( x = 2 \).**

      We want \( y = mx + b \). \( m = f'(2) = 6 \cdot 2 - 2 = 10 \), so \( y = 10x + b \).

      When \( x = 2 \), \( y = f(2) = 3 \cdot 2^2 - 2 \cdot 2 = 8 \).

      Thus, \( 8 = 10 \cdot 2 + b \), so \( b = -12 \) and we have \( y = 10x - 12 \).

   (d) **How would the derivative of \( g(x) = f(x) + 5 \) compare to \( f'(x) \)?**

      The graph of \( y = f(x) + 5 \) is the graph of \( y = f(x) \) shifted vertically by 5 units, but this has no effect on the slope of the graph, so \( g'(x) = f'(x) \).
(e) **How would the derivative of** $h(x) = 5f(x)$ **compare to** $f'(x)$?

The graph of $y = 5f(x)$ is the graph of $y = f(x)$ stretched vertically by a factor of 5; this also results in slopes that are 5 times greater at any given $x$-value. Thus, $h'(x) = 5f'(x)$.

Note that we get the same result by considering our derivative rule $\frac{d}{dx}[kf(x)] = kf'(x)$ where $k = 5$.

4. Given the graph of $f$, sketch a graph of $f'$ and a graph of $F$, an antiderivative of $f$ such that $F(0) = -2$.

Note: The concave up portion in the middle of the graph of $f$ is a perfect parabola, so its derivative ($f'$) is linear; since you don’t know the equation for $f$, your graph of $f'$ may be concave up/down there.
5. Shown below is a graph of $f'$ on its entire domain. The graph is NOT $f$.

At which $x$-value(s)

(a) does $f$ have a stationary point? $c, e, k$
(b) does $f$ have a local max? $e$
(c) does $f$ have a local min? $c$

(d) does $f'$ have a stationary point? $d, i, k$
(e) does $f'$ have a local max? $d, k$
(f) does $f'$ have a local min? $i$

(g) is $f$ greatest? $a$
(h) is $f$ least? $m$
(i) is $f'$ greatest? $d$
(j) is $f'$ least? $a$
(k) is $f''$ greatest? $b$
(l) is $f''$ least? $g$

On what interval(s)* is

(a) $f$ increasing? $c$ to $e$
(b) $f$ decreasing? $a$ to $c$ and $e$ to $m$

(c) $f'$ increasing? $a$ to $d$ and $i$ to $k$
(d) $f'$ decreasing? $d$ to $i$ and $k$ to $m$
(e) $f$ concave up? $a$ to $d$ and $i$ to $k$
(f) $f$ concave down? $d$ to $i$ and $k$ to $m$

* Whether to include the endpoints of these intervals will depend on your instructor’s definitions.

<table>
<thead>
<tr>
<th>$x$</th>
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6. Suppose that $T(t)$ gives the temperature in Lewiston as a function of time. In each of the following situations, determine if the signs of $T$, $T'$, and $T''$ are positive, negative, zero, or unknown.

(a) The temperature is 60 degrees and falling steadily.
   The temperature is 60, so we know $T$ is positive.
   The temperature is falling, so we know $T'$ is negative.
   The temperature is falling steadily, so we know the graph is linear, and $T''$ is zero.

(b) The temperature is rising more and more slowly.
   We don’t know whether the temperature is above or below zero, so the sign of $T$ is unknown.
   The temperature is rising, so we know $T'$ is positive.
   The temperature is rising more and more slowly, so we know the graph of $T$ is concave down, and $T''$ is negative.

(c) The temperature is $-5$ degrees and rising.
   The temperature is $-5$, so we know $T$ is negative.
   The temperature is rising, so we know $T'$ is positive.
   We don’t know the concavity of the graph of $T$, so the sign of $T''$ is unknown.

7. The table below gives some values for a function $f(x)$ whose derivative exists at all $x$. 

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10. **Solve the IVP (initial value problem)**

   $1 = x^3 - y'(x)$ if $y(2) = 13.$

   We begin by isolating $y'(x)$. This gives $y'(x) = -1 + x^3$.

   Next we find the antiderivative of $y'(x)$: $y(x) = -x + \frac{x^4}{4} + C$.

   Now we plug in the 2 and the 13 to find the value of $C$.

   $13 = -2 + \frac{2^4}{4} + C$

   $13 = -2 + 4 + C$

   $11 = C$

   So, the solution to this IVP is $y(x) = -x + \frac{x^4}{4} + 11.$

8. **Find the derivatives of the following.**

   (a) $y = 2 + 3x + x^4 + 5x^6$

   $y' = 3 + 4x^3 + 30x^5$

   (b) $y = \sqrt{x} + \frac{1}{x^6} + \frac{x}{6} + \frac{\pi}{6} + 6^{1/2}$

   First, rewrite $y$ to make it easier to apply our derivative rules:

   $y = x^{1/6} + x^{-6} + \frac{1}{6} \cdot x + 6x^{-1} + \frac{\pi}{6} + 6^{1/2}$

   $y' = \frac{1}{6}x^{-5/6} + (-6)x^{-7} + \frac{1}{6} + (6)(-1)(x^{-2}) + 0 + 0$

   If necessary, we can rewrite this using exponent rules: $x^{-n} = 1/x^n$ and $x^{m/n} = \sqrt[n]{x^m}$.

   $y' = \frac{1}{6\sqrt[6]{x^5}} - \frac{6}{x^7} + \frac{1}{6} - \frac{6}{x^2}$

9. **Find antiderivatives of the following.**

   (a) $y = \pi + 3x^2$

   antiderivative $= \pi x + x^3 + C$

   (b) $y = 4x^5 - \frac{1}{x^2} = 4x^5 - x^{-6}$

   antiderivative $= \frac{4x^6}{6} - x^{-5} + C = \frac{2x^6}{3} + \frac{1}{5x^5} + C$

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