Mathematics 106  
Exam I  
February 5, 2010

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<th>Problem</th>
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You must show all work to receive credit.  
No electronic devices other than calculators are permitted.  
Give exact answers (such as \( \ln 5 \) or \( e^2 \)) unless requested otherwise.
1. Consider the IVP \( y' = e^{-t}, \ y(0) = 0. \)

(a) Estimate \( y(3) \) using 6 Euler Steps.

(b) Is the estimate in part (a) an overestimate or an underestimate to the exact answer? Justify your answer (without appealing to (c)).

(c) Using separation of variables, provide an exact solution for this IVP.
2. Compute the exact arc length of \( f(x) = \frac{3}{4} x^{3/2} \) on the interval (0, 7).

3. A bucket that weighs 50 lbs when filled with water is lifted from the bottom of a well that is 100 ft deep. The bucket has a hole in it so it weighs only 25 lbs when it reaches the top of the well. Assume the water leaks out at a constant rate and the rope weighs .5 lb/ft. Set up but do not evaluate an integral whose value is the work required to lift the bucket from the bottom of the well to the top.
4. Let \( I = \int_{0}^{2} xe^{x^2} \, dx \).

(a) Evaluate the integral.

(b) Will \( L_4 \) be an overestimate or an underestimate? Why?

(c) Will \( M_4 \) be an overestimate or an underestimate? Why?

(d) Compute \( S_8 \) and \(|I - S_8|\)
5. Consider \( f(x) = \frac{x^2}{4} \) and \( g(x) = x \).
   
   (a) Draw the graphs \( g(x) \) and \( f(x) \).

   (b) Find the area between the two functions.

   (c) Find the volume created by revolving the area between the functions around the \( x \)-axis.

   (d) Set up the integral necessary to find the volume created by revolving the area between the function around the line \( y = -3 \).
6. Let \( I = \int_0^2 \frac{9}{130} x^{13/3} \, dx \).

(a) Recall \(|I - L_n| \leq \frac{B_1(b - a)^2}{2n}\). Provide the number of partitions necessary so that the error for \( L_n \) of \( I \) is less than .005.

(b) Recall \(|I - T_n| \leq \frac{B_2(b - a)^3}{12n^2}\). Provide the number of partitions necessary so that the error for \( T_n \) of \( I \) is less than .005.

(c) Recall \(|I - S_n| \leq \frac{B_4(b - a)^5}{180n^3}\). Provide the number of partitions necessary so that the error for \( S_n \) of \( I \) is less than .005.