February 5, 2010                                      Mathematics 206                                               Mr. Haines
Mathematics 206                                      Multi variable Calculus
Examination #1                                       

(15) I. If $\mathbf{a} = \mathbf{i} + \mathbf{j}$ and $\mathbf{b} = \mathbf{i} - 2\mathbf{j}$, compute these:

A. $\mathbf{a} \cdot \mathbf{b} =$

B. $\|\mathbf{b}\| =$

C. $\text{comp}_\mathbf{b}\mathbf{a} =$

D. $\text{proj}_\mathbf{b}\mathbf{a} =$
(5) II. Give an equation of the plane containing the points (1, 2, 3) and (3, 6, 7) and with normal \(2i - j\).

(10) III. Give examples of the following sets in \(\mathbb{R}^2\)

A. A set that is open and bounded.

B. A set that is open and not bounded.
(10) IV. \( A(t) = \left( 1 + t, t^2, \frac{1}{t} \right) \) with \( t \geq 1 \) is a path in \( \mathbb{R}^3 \).

A. Calculate \( A'(t) \), the derivative of \( A(t) \).

B. Give an equation of the tangent line to this path at the point where \( t = 1 \).

(5) V. Identify in words the surface whose equation is \( x^2 - y^2 - z^2 - 1 = 0 \).
VI. Suppose \( \mathbf{a} \) is a vector with tail at the point \((1,2,3)\) and head at the point \((3,5,5)\). Give a unit vector that is perpendicular to \( \mathbf{a} \).

VII. Compute the area of the parallelogram in \( \mathbb{R}^2 \) with vertices \((1,1)\), \((5,7)\), \((4,5)\), and \((2,3)\).
(5) VIII. The plane $P$ has coordinate equation $2x + 3y + z = 5$.

Give an equation for any line lying in $P$: 

(10) IX. Give examples of:

A. Two unit vectors in $\mathbb{R}^3$ that are perpendicular.

B. Equations of two distinct parallel planes.
(15) X. If \( f(x, y) = x \sin y \)

A. \( \frac{\partial f}{\partial x} (x, y) = \)

B. \( \frac{\partial f}{\partial y} (x, y) = \)

C. \( \frac{\partial^2 f}{\partial y \partial x} (x, y) = \)

D. \( \frac{\partial^2 f}{\partial x \partial y} (x, y) = \)
XI. For the quadratic form

\[ p(x,y,z) = -x^2 - 2y^2 - 5z^2 - 2xz \, , \]

A. Give a symmetric matrix \( S \) that is the matrix of this quadratic form.

B. By taking determinants and using Sylvester’s Theorem, determine if \( p \) is positive definite, negative definite, indefinite, or none of these.

XII. A student says that any three points in \( \mathbb{R}^3 \) determine a plane. She wants to find the equation of the plane that contains the points \( (1, 1, 3), (1, 0, 4), \) and \( (1, -1, 5) \). She knows she needs to find a normal to the plane, but has trouble computing it. Why?