Question 1) Evaluate the following integrals using the method of substitution.

a) \[ \int \frac{\cos x}{\sin^2 x} \, dx. \]

b) \[ \int_0^1 \frac{e^{-2x}}{\sqrt{1+e^{-2x}}} \, dx. \]
Question 2) Set up, but do not evaluate, the integral for the volume of the solid formed when the region bounded by \( y = e^x, y = 0, x = 1 \) and \( x = 0 \) is revolved around the \( y \)-axis.
Question 3) Consider the initial value problem (IVP) \( y' = xy^2 \) with \( y(1) = 1 \). Use separation of variables to find the solution to the IVP.
Question 4) Let $I = \int_0^2 3x^3 \, dx$.

a) Compute the approximation $T_5$ for the integral $I$.

b) How many subdivisions are required to obtain a midpoint sum approximation with error at most $1/1000$? Recall that the error bound estimates for midpoint sums may be determined using

$$|I - M_n| \leq \frac{K_2(b - a)^3}{24n^2}.$$
Question 5) Consider the IVP \( y' = y + t \) with \( y(0) = 1 \). Estimate \( y(2) \) using 4 Euler steps.
Question 6) A bucket that weighs 80 lb when filled with water is lifted from the bottom of a well that is 75 ft deep. However, the bucket has a hole in it, and so the bucket weighs only 40 lb when it reaches the top of the well. Suppose that water leaks from the bucket at a constant rate and that the rope weighs 0.65 lb/ft. Find the work required to lift the bucket from the bottom of the well to the top.