Correct answers accompanied by incorrect or incomplete work will not receive full credit. Please write your final answer in the space provided. Good Luck!

Formulas:

\[
\begin{align*}
|I - L_n| & \leq \frac{K_1(b-a)^2}{2n} \\
|I - R_n| & \leq \frac{K_1(b-a)^2}{2n} \\
|I - T_n| & \leq \frac{K_2(b-a)^3}{12n^2} \\
|I - M_n| & \leq \frac{K_2(b-a)^3}{24n^2}
\end{align*}
\]

\[
l = \int_a^b \left( \sqrt{(f'(x))^2 + 1} \right) \, dx
\]

1. (10 points) The table below indicates \( p(t) \) the rate of population growth (or decline) of a certain bacteria (in cells per minute). The net change in population from the time \( t = 0 \) minutes to time \( t = 5 \) minutes is \( I = \int_0^5 p(t) \, dt \). Use the Trapezoid Rule with 5 intervals (i.e., \( n = 5 \)) to estimate \( I \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
<th>4.5</th>
<th>5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p(t) )</td>
<td>9.7</td>
<td>9.0</td>
<td>8.0</td>
<td>6.7</td>
<td>5.1</td>
<td>3.3</td>
<td>1.4</td>
<td>-0.5</td>
<td>-2.5</td>
<td>-4.4</td>
<td>-6.1</td>
</tr>
</tbody>
</table>

\[
\Delta x = \frac{5 - 0}{5} = 1
\]

\[
\frac{1}{2} \left( 13.8 \right) = 13.8 \text{ cells}
\]

\[
\frac{1}{2} \left( 13.8 \right) = 13.8
\]
2. (14 points) Find real numbers $a$ and $b$ so that equality holds. Then evaluate the definite integral.

\[
\int_0^2 xe^{-x^2}dx = -\frac{1}{2} \int_a^b e^u du
\]

\[u = -x^2\]
\[du = -2x \, dx\]
\[\frac{-1}{2} \, du = x \, dx\]

\[
\begin{align*}
\int_0^2 xe^{-x^2}dx &= -\frac{1}{2} \int_0^2 e^u du \\
&= -\frac{1}{2} e^{-u^2} \bigg|_0^2 \\
&= -\frac{1}{2} e^{-4} + \frac{1}{2} e^0 = \frac{1}{2} - \frac{1}{2} e^4
\end{align*}
\]

\[a = 0\]
\[b = -4\]

\[
\text{Integral} = \frac{1}{2} - \frac{1}{2} e^4 = 4.908
\]

3. (10 points) Evaluate the indefinite integral.

\[
\int \frac{\sin x}{1 + \cos^2 x} \, dx
\]

\[u = \cos x\]
\[du = -\sin x \, dx\]

\[
\begin{align*}
\int \frac{-du}{1 + u^2} &= -\tan^{-1}(u) + C \\
&= -\tan^{-1}(\cos x) + C
\end{align*}
\]
4. (14 points) Let \( I = \int_a^b f(x) \, dx \), where \( f \) is positive and increasing over the interval \([a, b]\). Indicate whether, for all \( n \geq 1 \), the statement *must* be true, *cannot* be true, or *may* be true. *Explain your choice.*

(a) \( R_n \leq I \)

(b) \( T_n \leq I \)

(4a) cannot be true

\[ \text{b/c the fn is increasing we have a theorem that says } I \geq R_n \]

(4b) may be true

\[ \text{In the left graph } T_n \leq I \text{ b/c it is concave down. In the right graph } T_n \leq I \text{ b/c it is concave up.} \]

5. (10 points) Write the definite integral(s) that equals the length of the graph of \( y = x \sin x \) from \( x = -\pi \) to \( x = \pi/2 \). (Set up the integral only. Do NOT evaluate it.)

\[ \int_{-\pi}^{\pi/2} \sqrt{(x \cos x + \sin x)^2 + 1} \, dx \]

\[ y = x \sin x \]

\[ y' = x \cos x + \sin x \]
6. Let $R$ be the region bounded by $y = \frac{1}{x}$, $y = \frac{1}{4}x$, $x = 3$.

(a) (10 points) Set up the integral(s) that you would use to find the area of $R$.

$$\int_{0}^{\frac{1}{3}} \frac{1}{y} \, dx + \int_{2}^{3} \frac{1}{x} \, dx$$

(b) (10 points) Set up the integral(s) that you would use to find the volume of the solid rotated around the $y$-axis.

$$\pi \int_{0}^{\frac{1}{3}} \left[ 3^2 - (4y)^2 \right] \, dy + \pi \int_{\frac{1}{3}}^{\frac{1}{2}} \left[ \left( \frac{1}{y} \right)^2 - \left( 4y \right)^2 \right] \, dy$$

(c) (10 points) Set up the integral(s) that you would use to find the volume of the solid rotated around the line $y = -1$.

$$\pi \int_{0}^{\frac{1}{3}} \left[ (1 + \frac{1}{y} \cdot x)^2 - 1^2 \right] \, dx$$

$$+ \pi \int_{\frac{1}{3}}^{3} \left[ \left( 1 + \frac{1}{x} \right)^2 - 1^2 \right] \, dx$$

7. (2 points) Who do you think will win the SuperBowl this weekend? (Patriots or Giants?)

8. (10 points) Take home question. Pick up the question and directions as you leave.