Math 106: Review for Exam I

1. Find the following. [Substitution tip: usually let \( u = \) a function that’s “inside” another function, especially if \( du \) (possibly off by a multiplying constant) is also present in the integrand.]

   (a) \[ \int_{1}^{4} \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx \]

   (b) \[ \int_{\pi}^{2\pi} \cos^7(5x) \sin(5x) \, dx \]

   (c) \[ \int \frac{7x^2}{1 + x^6} \, dx \]

   (d) \[ \int_{6}^{10} x\sqrt{10 - x} \, dx \]

2. If \( f(x) \) is decreasing and concave up, put the following quantities in ascending order.

   \( L_{100}, R_{100}, T_{100}, M_{100}, \int_{a}^{b} f(x) \, dx \)

   What can you say with certainty about where \( S_{200} \) would fit into your list above?
3. Suppose $f(t)$ is the rate of change (in animals per month) of a population $P(t)$.

(a) What does $\int_{4}^{12} f(t) \, dt$ represent in this problem?

(b) Find the best possible left, right, midpoint, trapezoidal, and Simpson’s approximations to $\int_{4}^{12} f(t) \, dt$ given the data in the table below.

<table>
<thead>
<tr>
<th>$t$</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(t)$</td>
<td>15</td>
<td>11</td>
<td>8</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

4. Find bounds for each of the following errors if $I = \int_{2}^{7} \ln x \, dx$.

(a) $|I - L_{100}|$

(b) $|I - T_{100}|$

(c) $|I - M_{100}|$

5. Use Euler’s method with three steps on the differential equation $\frac{dy}{dt} = y - t$ to estimate $y(2.5)$ if $y(1) = 0$. 
6. Write integrals equal to
   (a) the arc length of $y = x^2$ on the interval $[1, 5]$

   (b) the area bounded by $y = x^2 - 8x + 24$ and $y = 3x$

7. Consider the region bounded by $y = \sqrt[3]{x}$, $y = 0$, and $x = 9$. Write an integral equal to the volume generated if this region is rotated about
   (a) the $x$-axis

   (b) the line $x = -1$

8. A pyramid has a square base 30 feet to a side and a height of 10 feet. Write integrals equal to
   (a) the volume of the pyramid

   (b) the work done in pumping all the fluid to a point 5 feet above the pyramid if the pyramid is filled to a height of 8 feet with water (62.4 pounds per cubic foot)