1. Let \( A = \begin{bmatrix} 2 & -4 & 1 & 8 \\ 5 & -10 & -1 & 13 \\ -3 & 6 & 1 & -7 \end{bmatrix} \).  
   1A) Let \( u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \). Use the “super-augmented” matrix technique developed in class to find any/all conditions on \( u_1 \), \( u_2 \), and \( u_3 \) that guarantee the matrix equation \( Ax = u \) is consistent. Show both the super-augmented matrix you use and its RREF.

   Consider \( \begin{bmatrix} 2 & -4 & 1 & 8 \\ 5 & -10 & -1 & 13 \\ -3 & 6 & 1 & -7 \end{bmatrix} \)

   Its RREF is \( \begin{bmatrix} 1 & -2 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \)

   The last row requires
   \[
   0 = u_1 - \frac{3}{2} u_2 - \frac{3}{2} u_3 \quad \text{or} \quad u_1 = \frac{3}{2} u_2 + \frac{3}{2} u_3
   \]

   in order for \( Ax = \hat{u} \) to be consistent.

1B) Show that the vector \( v = \begin{bmatrix} 1 \\ -8 \\ 6 \end{bmatrix} \) satisfies the condition(s) you found in (1A).

   \[ \begin{align*}
   1 &= \frac{5}{2}(8) + \frac{7}{2}(6) \\
   &= 5(4) + 7(3) \\
   &= -20 + 21 \\
   \end{align*} \]

1C) Find all the solutions of \( Ax = v \) and express them in parametric vector form.

   \[
   \begin{bmatrix}
   A & | & v \\
   \end{bmatrix} = \begin{bmatrix}
   2 & -4 & 1 & 8 \\
   5 & -10 & -1 & 13 \\
   -3 & 6 & 1 & -7 \\
   \end{bmatrix} \begin{bmatrix}
   1 \\
   -8 \\
   6 \\
   \end{bmatrix}
   \]

   has RREF \( \begin{bmatrix} 1 & -2 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \) which tells us the solution \( x \)

1D) What specific solution \( s = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} \) do you get for \( Ax = v \) if you set all the free variables in (1C) to 10?

   \[
   \begin{bmatrix}
   s_1 \\
   s_2 \\
   s_3 \\
   s_4 \\
   \end{bmatrix} = \begin{bmatrix}
   -1 + 20 + 30 \\
   10 \\
   -20 \\
   10 \\
   \end{bmatrix}
   \]

1E) Find \( As \) (do this product using your calculator and two of its matrices as shown in class) for the vector \( s \) in (1D). What do you get for \( As \)?

   Since \( s \) is a solution of \( A\hat{x} = \hat{v} \), we get \( A\hat{s} = \begin{bmatrix} -8 \\ 6 \end{bmatrix} \) (no need to use a calculator unless you'd like to verify this!)