Math 106: Review for Exam I

1. Find the following. [Substitution tip: usually let \( u \) = a function that’s “inside” another function, especially if \( du \) (possibly off by a multiplying constant) is also present in the integrand.]

(a) \( \int_{1}^{4} \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx \)

(b) \( \int_{\pi}^{2\pi} \cos^7(5x) \sin(5x) \, dx \)

(c) \( \int \frac{7x^2}{1 + x^6} \, dx \)

(d) \( \int_{6}^{10} x \sqrt{10 - x} \, dx \)
2. Suppose \( f(x) \) is decreasing and concave up.
   
   (a) Put the following quantities in ascending order.
   
   \[ L_{100}, R_{100}, T_{100}, M_{100}, \int_{a}^{b} f(x) \, dx \]
   
   (b) What can you say with certainty about where \( S_{200} \) would fit into your list above?

3. Suppose \( f(t) \) is the rate of change (in animals per month) of a population \( P(t) \).
   
   (a) What does \( \int_{4}^{12} f(t) \, dt \) represent in this problem?
   
   (b) Find the best possible left, right, midpoint, trapezoidal, and Simpson's approximations to \( \int_{4}^{12} f(t) \, dt \) given the data in the table below.

   \[
   \begin{array}{|c|c|c|c|c|c|}
   \hline
   t & 4 & 6 & 8 & 10 & 12 \\
   \hline
   f(t) & 15 & 11 & 8 & 4 & 3 \\
   \hline
   \end{array}
   \]

4. Find bounds for each of the following errors if \( I = \int_{2}^{7} \ln x \, dx \).
   
   (a) \( |I - L_{100}| \)
   
   (b) \( |I - T_{100}| \)
   
   (c) \( |I - M_{100}| \)
5. If \( I = \int_{2}^{7} \ln x \, dx \), how many subdivisions are required to obtain a trapezoidal sum approximation with error of at most \( 1/1,000,000 \)?

6. Solve the differential equation \( \frac{dy}{dx} = 2xy + 6x \) if the solution passes through \((0,5)\). [Students in the 9:30 section should omit this problem.]

7. Write integrals equal to
   
   (a) the arc length of \( y = x^2 \) on the interval \([1, 5]\)

   (b) the area bounded by \( y = x^2 - 8x + 24 \) and \( y = 3x \)
8. Consider the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 9$. Write an integral equal to the volume generated if this region is revolved about

(a) the $x$-axis

(b) the line $x = -1$